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A novel extension of Frantz–Nodvik laser-amplifier model: a computational study

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Abstract. The Frantz–Nodvik model was extended to propagate pulse through a double-pass laser amplifier and to study beam-interaction effects on the output-energy fluence and gain factor according to the distance between two beams. We also obtained the necessary approximation equations for the saturation state and small input pulse. The results show that minimum output energy and gain occur when two beams completely coincide. The comparison between numerical, analytical, and empirical results demonstrates a good correspondence.

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Key words: double-pass laser amplifier, Frantz–Nodvik theory, pulse dynamics

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1 Introduction

Increasing efficiency and decreasing amplifier levels are of great significance particularly in laser-joint systems, and an appropriate analytical model can successfully be used to optimize the design of such optical structures. In practice, double- or multi-pass amplifiers functionally amplify extremely low- and high-energy pulses [1–3]. Most analytical models that have been proposed for optical amplifiers are based on the Frantz–Nodvik theory [4]. In this model, only by assuming that the gain factor depends on the direction of laser beam propagation through the amplifying medium, the relationship between input- and output-energy fluxes is obtained according to the initial gain factor and saturation-energy flux. Modifying the gain factor, this model is used again for the second pass [5]. By providing a simple analysis and assuming that the initial distribution of gain is uniform throughout the active medium, the Frantz–Nodvik model leads to very good results for explaining the amplifier's behavior. When the laser pulse width (t_p)

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is much larger than the time necessary for the optical pulse to pass through the amplifier (τ) and the gain slightly changes during pulse pass due to pumping and under-drop mechanisms, the Frantz–Nodvik model that is independent of input pulse's temporal behavior and the laser active medium's type is valid and employed [6–9]. When the gain distribution is not uniform due to the amplifier's non-uniform pumping or the radial dependence of the distribution of input intensity, a special accuracy is required to employ the Frantz–Nodvik model [10–13]. In such instance, by assuming that the gain spatial distribution is known before the optical pulse enters the amplifier, the one pass version of this model can be used for every point inside the amplifier. The transverse distribution of output-beam intensity can be obtained by iterating this model. For consecutive passes and the assumption that $t_p \gg \tau$, the optical gain of each beam is affected by its neighboring beams' intensity. In this case, except for a special state in which the beams' propagation directions coincide [14], there is not any appropriate analytical model to describe how the output-energy flux depends on the distance between beams' propagation directions [15–20]. The only existing model of multi-pass amplifiers provides necessary calculations just by qualitative and approximate considerations concerning beams overlapping and regardless of passing beams' spatial position [21]. Hence, for appropriate initial and boundary conditions on the amplifier's input and output surfaces, the mass contrast and photon flux of each beam can be obtained only through numerically solving the position-dependent rate equations [22–25].

In this paper, for the first time, after solving the rate equations for a double-pass amplifier, we obtain the analytical solutions for the dependence of input-energy fluence on the radial-position-dependent optical gain and transmission factor between the first pass and second pass, and the fluence of output energy to gain for the known directions of input and output beams. The obtained results are consistent with the numerical and empirical results.

2 Position-dependent rate equations for double-pass amplifiers

When the input beam's temporal width is small enough compared with the upper level's lifetime and active medium's pumping time, the following simplified rate equations can be used to obtain the mass contrast between the active medium's upper and down levels ($N(r,z,t)$) and the photon density of first and second passes ($\Phi_1(r,z,t)$ and $\Phi_2(r,z,t)$) [10]

$$\frac{\partial}{\partial t}\Phi_1(r,z,t)+c\frac{\partial}{\partial z}\Phi_1(r,z,t)=N(r,z,t)c\sigma\Phi_1(r,z,t) \tag{1}$$

$$\frac{\partial}{\partial t}\Phi_2(r,z,t)-c\frac{\partial}{\partial z}\Phi_2(r,z,t)=N(r,z,t)c\sigma\Phi_2(r,z,t) \tag{2}$$

$$\frac{\partial}{\partial t}N(r,z,t)=-N(r,z,t)c\sigma[\Phi_1(r,z,t)+\Phi_2(r,z,t)] \tag{3}$$