Generation of three-atom W state via nonresonant Jaynes-Cummings model

Deng-Yu Zhang*, Xin-Wen Wang, Shi-Qing Tang, and Li-Jun Xie

Department of Physics and Electronic Information Science, Hengyang Normal University, Hengyang 421008 China

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Abstract. A simple scheme is presented for generating three-atom W state via nonresonant Jaynes-Cummings model. In the proposed protocol, it is injected two two-level atom initially prepared in the excited state and ground state through the cavity prepared in the vacuum state. The third two-level atom is prepared in the ground state and three two-level atom are injected into the second cavity prepared in the vacuum state. The experiment of the proposed scheme is feasible.

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Key words: two-level atom, cavity field, nonresonant interaction, W state

1 Introduction

Entanglement is not only the most striking feature of quantum mechanics, but also an important physical resource in quantum information processing (QIP), such as quantum teleportation, quantum dense coding, and quantum cryptography. Recently, multiparticle entanglement has attracted much interest due to its potential applications in QIP. In general, the more particles that can be entangled, the more clearly nonclassical effects are exhibited, and the more useful the states are for quantum applications [1-8]. Thus generation and manipulation of multipartite entangled states are very important tasks in QIP and have been attracting much attention. Dür et al have shown that there are two inequivalent classes of tripartite entanglement states, i.e., the W class and the GHZ class, under stochastic local operations and classical communications [9]. Zheng presented a scheme for the generation of the W state for trapped ions [10]. Zhong proposed an alternative scheme to generate four-photon W state via resonant cavity QED [11]. W type of entangled states have many interesting properties. Hence, the preparation of W state

*Corresponding author. Email address: dyzhang672@163.com (D.-Y. Zhang)
become a critical technique in QIP. In this paper, we propose a scheme for generation of three-atom W state via cavity QED. It is based on the nonresonant Jaynes-Cummings model of two and three identical atoms simultaneously interacting with single-mode cavity field in the vacuum state. The proposed scheme is simple and feasible because we only employ three two-level atoms and two single-mode cavities, and is easier to be realized in experiment.

2 Generation of W state via nonresonant Jaynes-Cummings model

We consider that \( N \) identical two-level atoms are simultaneously coupled to the same mode of cavity field, in which we are considering dipole-dipole coupling between atoms. The Hamiltonian of the system can be written as \((\hbar = 1)\)

\[
H = \omega_0 a^+ a + \sum_{i=1}^{N} \omega_i S_i^Z + \sum_{i=1}^{N} g_i (a^+ S_i^- + a S_i^+) + \sum_{i \neq j} \Omega_{ij} S_i^+ S_j^- , \tag{1}
\]

where \( a \) and \( a^+ \) are the annihilation and creation operators for the cavity field, \( S_i^Z, S_i^- \) and \( S_i^+ \) are the inversion, lowering and raising operators for the \( i \)-th atom, respectively, \( \omega_i \) is the \( i \)-th atomic transition frequency and \( \omega_0 \) is the field frequency, \( g_i \) is the \( i \)-th atom-field coupling constant and \( \Omega_{ij} \) is the strength of the dipole-dipole interaction between atom \( i \) and \( j \). We assume that each atomic transition frequency is highly detuned from cavity mode, i.e., the detuning \(|\Delta_i| = |\omega_i - \omega_0| \gg g_i\). For simplicity, \( N \) two-level atoms are assumed to be identical so that \( g_1 = g_2 = \cdots = g, \omega_1 = \omega_2 = \cdots = \omega, \Delta_1 = \Delta_2 = \cdots = \Delta \), and \( \Omega_{ij} = \Omega \). The effective Hamiltonian for such a system in the interaction picture is given by \([12, 13]\)

\[
H_{\text{eff}} = \sum_{i=1}^{N} \frac{2g^2}{\Delta} a^+ a S_i^Z + \Omega \sum_{i \neq j} S_i^+ S_j^- . \tag{2}
\]

Assume that atoms is initially in \( |\psi(0)\rangle_a \) and the cavity is initially in \( |\psi(0)\rangle_f \), then the initial state of the atoms-cavity field system is

\[
|\psi(0)\rangle_s = |\psi(0)\rangle_a \otimes |\psi(0)\rangle_f \tag{3}
\]

The evolution of the whole system is

\[
|\psi(t)\rangle = \exp(-iH_{\text{eff}}t) |\psi(0)\rangle_s \tag{4}
\]

For \( N = 3 \) in Eq. (2), we assume that two identical single-mode cavity fields are initially prepared in the vacuum state \( |0\rangle \), atom 3 is initially in \( |g\rangle_3 \). Atoms 1 and 2 are prepared in the state \( |eg\rangle_{12} \), which are sent through the first cavity simultaneously. After interaction time \( \tau_1 \), the state of system becomes \([13]\)

\[
|\psi(\tau_1)\rangle_{12s} = |\psi(\tau_1)\rangle_{12} = |\text{cos}(\Omega \tau_1)|eg\rangle_{12} - i \text{sin}(\Omega \tau_1)|ge\rangle_{12} \otimes |0\rangle_f \tag{5}
\]