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Relativistic theory of one– and two electron systems: valley of stability in the helium-like ions

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Abstract. A semi-relativistic total energy of the hydrogen-like ions is presented. The established expression taking only into account the dependence of the mass electron on the speed could be considered as a first correction of the Bohr's semi-classical formula. Comparison with relativistic total energy expression obtained from the Dirac's relativistic wave equation is made. In addition, the present relativistic theory of the hydrogen- like ions is extended to the helium isoelectronic series. It is shown that, for the ground state of two electron systems, the relativistic screening constant σ^{rel} decreases when increasing the nuclear charge up to $Z=5$. Beyond, σ^{rel} increases when increasing Z and, the plot $\sigma^{\text{rel}}=f(Z)$ is like a valley of stability where the bottom is occupied by the B^{3+}-helium-like ion. As a result, only He, Li^{+}, Be^{2+} and B^{3+} exist in the natural matter in low temperature. All the other helium-like positive-ions, such as C^{4+}, N^{5+}, O^{6+}, F^{7+}, Ne^{8+}, $\cdots$, can only exist in hot laboratory and astrophysical plasmas.

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Key words: screening constant, hydrogen-like ions, helium-like ions, valley of stability

1 Introduction

In the year 1913, Bohr managed to explain the spectrum of the hydrogen atom by an extension of Rutherford's planetary atomic model (1911). In the Bohr's model, the negatively charged electron revolves about the positively charged atomic nucleus because of the attractive electrostatic force according to Coulomb's law. On the basis of this classical atomic model, Bohr expresses total energy of the hydrogen atom considering the mass electron as constant, independent then with his velocity. But, since 1905, Einstein develops the theory of relativity and shows that mass of rapid elementary particles varies with their speed. As classical theory is not the framework for interpreting atoms, relativistic corrections of Bohr's formula may be

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done just after 1913. But, these relativistic corrections have been done only in the year 1916 by Sommerfeld (in the framework of the elliptical orbits model) and after in the year 1926 by use of the relativistic wave equation established by Dirac after the discovery of the spin electron (1925) by Uhlenberg and Goudsmith. However, it would be very interesting to make relativistic correction of the Bohr's semi-classical formula before 1916, considering only the dependence of the mass electron on the electron's speed as explained by Einstein. This would permit to interpret theoretically experimental studies who indicate that [1], due to the variation of the mass electron with the speed, the relativistic total energy levels of the hydrogen atom are lowest than the non perturbated total energy levels. Taking after into account the spin electron, one may explain clearly the contribution of the spin in the relativistic effects on the hydrogen-like ions energy-levels. This paper is prepared in the intention to show that, the relativistic effects due to the variation of the electron mass with the speed on the hydrogen isoelectronic sequence energy levels can be put into evidence separately with that due to the spin. In the Dirac's theory, this separation is not possible, as his relativistic wave equation is constructed by considering simultaneously the spin electron and the variation of the mass electron with the speed. In addition, the presented relativistic theory of the hydrogen-like ions is extended to the helium isoelectronic series. It is shown that, for the ground state of two electron systems, the relativistic screening constant σ^{rel} decreases when increasing the nuclear charge up to $Z=5$.

Beyond, σ^{rel} increases when increasing Z, and the plot $\sigma^{\mathrm{rel}}=f(Z)$ is like a valley of stability where the bottom is occupied by the B^{3+}-helium-like ion. As a result, only He, Li^{+}, Be^{2+} and B^{3+} exist in the natural matter in low temperature (LiO, $Be(OH)_2$, and B_2O_3 for example). All the other helium-like positive-ions, such as C^{4+}, N^{5+}, O^{6+}, F^{7+}, Ne^{8+}, $\cdots$, can only exist in hot laboratory and astrophysical plasmas.

2 Theory

2.1 Bohr's semi-classical expression of hydrogen-like ions total energy

In the view point of the Bohr's model, the quantized energy of the hydrogen-like ions is

$$E_n=-\frac{Z^2\alpha^2mc^2}{2n^2} \tag{1}$$

where α denotes the fine structure constant and mc^2 the rest energy of the electron.

On the other hand in the framework of the Bohr's theory, the total energy E_n and the kinetic energy E_c satisfy the relation

$$E_n=-E_c.$$

That is to say using Eq. (1)

$$E_n=-\frac{Z^2\alpha^2mc^2}{2n^2}=-\frac{1}{2}mv_n^2.$$