

## Two kinds of entangled coherent states and their nonclassical effects

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**Abstract.** As for two kinds of entangled coherent states, we have studied the relationship between the entanglement and the nonclassical effects; we calculate their entanglement by the concurrence and their nonclassical effects, such as squeezing and antiquating. We find that the entanglement always corresponds with one of squeezing and antiquating and the larger a nonclassical effect is, the stronger entanglement is. The result shows the entanglement has a deep relationship with the nonclassical effects.

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**Key words:** entangled coherent states, nonclassical effects, concurrence

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### 1 Introduction

Since 1935, entanglement has been recognized as one of the most puzzling features of quantum mechanics [1, 2]. However, it is nowadays a widespread opinion that it also represents a fundamental resource for many quantum information protocols. Therefore, entanglement deserves to be analyzed in all respects. After the first experiment on quantum teleportation [3] and other quantum information processes using two-mode squeezing states [4, 5], continuous variable systems have aroused great interest in the separable properties. So far, most theoretical and experimental work has focused on the entanglement properties of Gaussian states. For Gaussian states, the necessary and sufficient inseparability criterion has been fully developed [6, 7], Inseparability Criteria for Continuous Bipartite Quantum States has also been developed [8, 9]. Using the total variance of a pair of Einstein-Podolsky-Roses type operators introduced by Duan *et al.* [6], generalized EPS entangled states (GEES) has been obtained and it has been proved that a state must be the two-mode squeezing state if the state is a GEES whether it is Gaussian or not and whether it is pure or not. However, there are some

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entangled coherent states which have entanglement, but do not have squeezing. Whether this kind of entanglement corresponds with some nonclassical effects of two-mode fields? [10]

In this paper, we analyze two kinds of entangled coherent states and calculate their entanglement by concurrence and their nonclassical effects, such as squeezing and anti-squeezing, and find that the entanglement always follows one of squeezing and anti-squeezing and the entanglement increases with the increase of a nonclassical effect. It shows the entanglement has a deep relationship with the nonclassical effects.

## 2 Squeezing and anti-squeezing of two kinds entangled coherent states

Firstly, we consider the following bipartite entangled coherent states [11]

$$|\psi\rangle = \mu|\alpha, \alpha\rangle + \nu|-\alpha, -\alpha\rangle, \quad (1)$$

where  $|\alpha\rangle, |-\alpha\rangle$  are normalized states of system 1 and similarly  $|\alpha\rangle, |-\alpha\rangle$  are states of system 2 with complex  $\mu$  and  $\nu$ , after normalization, the bipartite states  $|\psi\rangle$  are given by

$$|\psi\rangle = \frac{1}{N} \left( \mu|\alpha, \alpha\rangle + \nu|-\alpha, -\alpha\rangle \right), \quad (2)$$

where  $N^2 = |\mu|^2 + |\nu|^2 + (\mu^*\nu + \mu\nu^*)e^{-4R^2}$ . The two non-orthogonal states  $|\alpha\rangle, |-\alpha\rangle$  are assumed to be linearly independent and span a two-dimensional subspace of the Hilbert space, and then we choose an orthogonal basis  $\{|0\rangle_i, |1\rangle_i\}$  ( $i=1,2$ ) [12]

$$\begin{aligned} |0\rangle_1 &= |\alpha\rangle_1, |1\rangle_1 = \frac{1}{\sqrt{1-P^2}} \left( |-\alpha\rangle_1 - P|\alpha\rangle_1 \right), |0\rangle_2 \\ &= |-\alpha\rangle_2, |1\rangle_2 = \frac{1}{\sqrt{1-P^2}} \left( |\alpha\rangle_2 - P|-\alpha\rangle_2 \right), \end{aligned} \quad (3)$$

with  $P = e^{-2R^2}$ .

Under these bases, the entangled states  $|\psi\rangle$  can be rewritten as

$$|\psi\rangle = \frac{1}{N} \left( (\mu P + \nu P)|00\rangle + \mu\sqrt{1-P^2}|01\rangle + \nu\sqrt{1-P^2}|10\rangle \right), \quad (4)$$

which shows that the general entangled non-orthogonal state is considered as a state of two logical qubits, then it is straightforward to obtain the reduced density matrix  $\rho_1$  and two eigenvalues of  $\rho_1$  are given by [13]

$$\lambda_{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \frac{4|\mu\nu|^2}{N^4} (1-P^2)^2}, \quad (5)$$