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Entropy squeezing of a three-level atom interacting with a cavity field

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Abstract. We study the field entropy squeezing as a measure of the entanglement in a three-level system interacting with a cavity field. Numerical calculations under current experimental conditions are performed and it is found that the initial state setting and atom-field coupling present changes of the general features of the field entropy squeezing dramatically.

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Key words: entropy squeezing, three-level atom, entanglement

1 Introduction

Great progress has recently been made in quantum information theory [1]. Also, entropy becomes a fundamental quantity to describe not only uncertainty or chaos of a system but also information carried by the system [2]. Compared to the long history of the theoretical understanding of entropy and entanglement of atom-field systems extending over many decades [3], intensive experimental investigations started only recently involving different systems [4].

To identify the fundamentally inequivalent ways quantum systems can be entangled is a major goal of quantum information theory [4]. It might be thought that there is nothing new to be said about bipartite entanglement if the shared state is pure, but in a

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recent paper [5] it has been shown that exact coherence of the atom is in general never regained for a two-level model with a general initial pure quantum state of the radiation field. Also, it has been shown that the purification of the atomic state is actually independent of the nature of the initial pure state of the radiation field. An analysis of both analytical and numerical investigations of the process of atomic information entropy in the three-level systems has been presented [6].

Our motivation is to discuss the entropy squeezing from another point of view by considering the entropy squeezing for the field instead of the atom. Using an appropriate representation, an explicit expression for entropy squeezing when the system starts from a mixed state is derived. The physical situation which we shall refer to, belongs to the experimental domains of cavity quantum electrodynamics.

2 The model and its solution

We start by devoting this section to a brief discussion on the 3-level atom [7,8] being it the model describing the interaction between a single multi-level atom and a quantized cavity field. To set the stage, we first begin by describing the multilevel-atom model. Therefore, the physical system on which we focus is an 3-level. The atom interacts with a high Q-cavity which sustain a single cavity field with frequencies Ω. We denote by $\hat{a}$ and $\hat{a}^\dagger$ the annihilation and creation operators for the field mode, and ω_j is the frequency associated with the level of the atom. Therefore in the rotating wave approximation we can cast the Hamiltonian of the system in the form [8] ($\hbar=1$)

$$\hat{H}=\hat{H}_0+\hat{H}_1, \tag{1}$$

where the Hamiltonian for the interacting system $\hat{H}_0$ is given by

$$\hat{H}_0=\Omega\hat{a}^\dagger\hat{a}+\sum_{i=1,2,3}\omega_i|i\rangle\langle i|.$$

The interaction Hamiltonian between the atomic system and the cavity field is given by

$$\hat{H}_1=\sum_{j=1}^{2}\lambda_j(\hat{S}_{1,j+1}\hat{a}+h.c.).$$

The transition in the 3-level atom is characterized by the coupling λ_j. The operator $\widehat{S}_{ii}$ describes the atomic population of level $|i\rangle_A$ with energy $\omega_i,(i=1,2,3)$ and the operator $\widehat{S}_{ij}=|i\rangle\langle j|,(i\neq j)$ describes the transition from level $|i\rangle_A$ to level $|j\rangle_A$.

We have applied the rotating wave approximation discarding the rapidly oscillating terms and selecting the terms that oscillate with minimum frequency [9]. The resulting