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Bound state solutions of the s-wave Klein-Gordon equation with position dependent mass for exponential potential

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Abstract. Bound state solutions of the s-wave Klein-Gordon equation with spatially dependent exponential-type mass for exponential-type scalar and vector potential are studied by using the Nikiforov-Uvarov method. The wave functions of the system are taken on the form of the Laguerre polynomials and the energy spectra of the system are discussed. In limit of constant mass, the wave functions and energy eigenvalues are in good agreement with the results previously.

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Key words: Klein-Gordon equation, Bound state solution, position dependent mass, exponential potential

1 Introduction

When a particle is in a strong potential field, the relativistic effect must be considered, which gives the correction for non-relativistic quantum mechanics. Taking the relativistic effect into account, one can apply the Klein-Gordon equation to the treatment of a zero-spin particle and apply the Dirac equation to that of a 1/2-spin particle. In fact, the problem of exact solutions of the Klein-Gordon equation for a number of special potential has also been a line of great interest in the recent years [1–7]. For example, some authors assumed that the scalar potential is equal to the vector potential and obtained the exact solutions of the klein-Gordon equation with some typical potential by using different methods. These investigations include the harmonic oscillator [8], the triaxial and axially deformed harmonic oscillators potential [9], Eckart potential [10, 11], Woods-Saxon potential [12], pseudoharmonic oscillator [13], ring-shaped Kratzer-type potential [14], ring-shaped non-spherical oscillator [15],

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double ring-shaped oscillator [16], Hartmann potential [17, 18], Rosen-Morse-type potential [19], generalized symmetrical double-well potential [20], Scarf-type potential [21], etc. These methods include the standard method, supersymmetry quantum mechanics [10], the Nikiforov-Uvarov (NU) method [12, 22–24], and others.

On the other hand, the concept of the position dependent mass in the quantum mechanical systems has also attracted a lot of attention and inspired intense research activities. They are indeed very useful and have been widely used in many different fields, such as semiconductor physics [25], quantum wells and quantum dots [26], He clusters [27],quantum liquids [28] and semicondector heterostructures [29], etc. In recent years, the solutions of the nonrelativistic wave equation with position dependent mass have been a line of great interest [30–34] but there are only few contributions that give the solution of the relativistic wave equation with position dependent mass in the quantum mechanics. Alhaidari [35] studied the exact solution of the Dirac equation with position dependent mass in the Coulomb field. Vakarchuk [36] investigated the Kepler problem in Dirac theory for a particle whose potential and mass are inversely proportional to the distance from the force center. Jia *et al.* investigated the approximately solution of the one-dimensional Dirac equations with spatially dependent mass for the generalized Hulthen potential [37]. Jia and Souza Dutra [38] considered position-dependent effective mass Dirac equations with PT and non-PT symmetric potential. In Ref. [39], Souza Dutra and Jia investigated the exact solution of the one-dimensional Klein-Gordon equation with spatially dependent mass for the inversely linear potential. Here we intend to study the one-dimensional Klein-Gordon equation for the exponential potential with an exponentially spatially dependent mass. We solve the equation by using the Nikiforov-Uvarov method [40] and discuss the limit of the constant-mass. The organization of this paper consists of three sections: In Section 2, we review the Nikiforov-Uvarov method briefly. Section 3 is devoted to the analytic bounded solutions of the Klein-Gordon equation for this quantum system by the NU method. Finally, the relevant results are discussed in Section 4.

2 Nikiforov-Uvarov method

The NU method is based on solving the second-order linear differential equation by reducing to a generalized equation of hypergeometric type.The NU method has been used to solve the Schrodinger,Dirac and Klein-Gordon wave equations for certain kind of potential [41]. In this method, the second-order differential equation can be written in the following form

$$\psi(s)'' + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0, \tag{1}$$

where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at most second degree, and $\tilde{\tau}(s)$ is a first-degree polynomial. In order to find a particular solution to Eq. (1), we use the following transformed

$$\psi(s) = \phi(s)y(s). \tag{2}$$