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COUPLING OF VISCOUS AND POTENTIAL FLOW MODELS WITH FREE SURFACE FOR NEAR AND FAR FIELD WAVE PROPAGATION

YI ZHANG, MAŁGORZATA PESZYŃSKA, AND SOLOMON C. YIM

Abstract. A non–overlapping domain decomposition (DD) method is used to solve a heterogeneous flow model which combines viscous flow and potential flow. Finite element method (FEM) and boundary element method (BEM) approximate the solutions to Navier–Stokes equations in the viscous flow subdomain and to Laplace equation in the potential flow subdomain, respectively. At the interface, the matching conditions involve pressure and velocity, and Bernoulli's equation gives an ordinary differential equation (ODE) defined on the interface. Algebraic formulations of the iterative schemes to solve the coupled problem are developed, and both explicit and implicit schemes can be constructed following the strategy of the Dirichlet–Neumann (D–N) method. Numerical examples using the explicit scheme implementation are reported and compared against previous experimental and/or numerical results.

Key words. free surface flows, finite element method, boundary element method, heterogeneous domain decomposition

1. Introduction

In this paper we discuss a multiphysics flow model for wave propagation with a free surface, and we apply a heterogeneous non-overlapping domain decomposition method in which different fluid models are coupled across an interface. In the model described here the flow domain is decomposed into near field and far field subdomains in which the viscous and potential flow models are applied, respectively. The coupling of models occurs across the interface between the two subdomains, and the form of the interface equations and their discretization is the crux of the coupled model. We refer to [34] for general nomenclature on domain decomposition methods.

The comprehensive model for viscous flow and wave propagation is that of Naver–Stokes equations (NSE) with free surface tracking. Since NSE are computationally complex, it is not feasible to use the viscous flow model in large flow domains where the waves propagate. On the other hand, the potential flow (PF) model is simple, and, with boundary element formulation and code optimization, its solver has achieved low complexity. The idea to couple the viscous (NSE) model with potential flow (PF) takes advantage of the relatively low computational cost of the PF compared to that of NSE, and of the ability of the potential flow model to approximate physical flow over wide ranges under certain conditions. The latter is true in the far-field (away from obstacles such as rigid or flexible structures) where the effects of viscosity and vortices can be neglected. See Figure 1 for the general idea.

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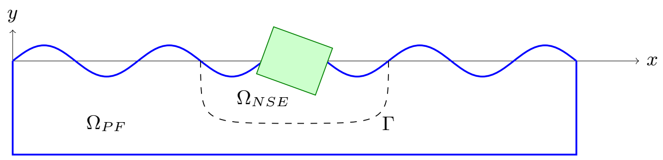

FIGURE 1. The flow domain for modeling water wave dynamics. Modeling of flow coupling with the structure dynamics will be considered in future work

More generally, heterogeneous model coupling is used when there is inherently different flow physics across interfaces, e.g., as in Fluid-Structure Interaction (FSI) where the fluid flow equations are coupled with solid mechanics models for structure deformation [33], or when one model can serve as an approximation to another, e.g., as in single–phase fluid flow approximating black-oil model in fully saturated zones of subsurface [30]. In the context of wave and fluid dynamics near structures, NSE are used to reproduce fine-scale drag, viscosity, vorticity, and turbulence near the structures, whereas PF is used for far field flow modeling, see coupled aerodynamics and hydrodynamics problems in [7, 8, 27, 37, 25]. More generally, heterogeneous domain decomposition methods were dicussed in [34, Chap 8], and specifically for flow models in [13, 36, 19, 20, 16, 17], and for coupled porous media flow and surface and porous media models in [30, 42, 14]. The difficulties with interface equations arising from incompatibility of variables, fluxes, and stresses are ubiquitous; see, e.g., [19, 20, 14].

The heterogeneous coupled PF-NSE model poses several challenges. First, i) the fact that PF is an approximation to NSE makes the formulation of a proper interface model difficult. This is exacerbated by ii) the numerical difficulties associated with coupling of a traditional grid-based discretization for NSE with the boundary element formulation of PF. Finally, iii) the treatment of the free surface boundaries and especially on the interface has to be properly incorporated.

The modeling challenges i) are due to an inherent incompatibility of some equations in PF and NSE, and involve the pressure, potential, and velocity variables. In particular, since usually the governing equation for the PF is based solely on mass conservation, the flow dynamics is missing and should be recovered using Bernoulli's equation. A stationary PF-NSE coupled model without free surface was found well-posed mathematically in [19], but the associated modeling difficulties in this static case were pointed out. A full nonstationary model coupling PF and NSE with free surface considered in [27] assigned PF to the bottom portion of flow field, and NSE to the top part, thus the fixed artificial interface between PF and NSE subdomains does not intersect the free surface, which simplifies handling ii) and iii). The interface equations in [27] involve the Bernoulli's equation, i.e., an ODE on interface, which is discretized with a Runge–Kutta solver, and the results reported in [27] are promising. However, since wave generation and dissipation boundaries are commonly required for free surface flow models (see, e.g., [26]), the vertical alignment of the subdomains (see Figure 2) requires such boundaries and appropriate computational modules for both subdomains and solvers. Moreover, the NSE subdomain in [27] has the same horizontal extension as the whole flow domain, thus it includes the upper part of the far field, increasing the computational cost.