INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 2, Number 4, Pages 298–314

SYMPLECTIC ANALYSIS FOR THE WAVE PROPAGATION PROPERTIES OF CONVENTIONAL AND AUXETIC CELLULAR STRUCTURES

XIUHUI HOU, ZICHEN DENG, AND JIAXI ZHOU

Abstract. Based on the structure-preserving characteristics of the symplectic algorithm, the wave propagation problem of auxetic cellular structures is analyzed and compared with conventional cellular materials. The dispersion relations along the first Brillouin zone boundary and the contour plots of phase constant surfaces are obtained using the finite element method. Numerical results reveal the superiority of auxetic re-entrant honeycombs in sound reduction applications compared with conventional hexagon lattices. Band structures of the chiral honeycomb are developed to illustrate the unique feature of the symplectic algorithm at higher frequencies calculation. The highly-directional wave propagation properties of auxetic cellular structures are also analyzed, which will provide invaluable guidelines for the future application of auxetic cellular structures in sound insulation.

Key words. Symplectic algorithm, Auxetic cellular structures, Wave propagation, and Chiral honeycombs.

1. Introduction

Cellular structures, as one kind of periodic structures, are capable of attenuating elastic waves over certain frequency bands, thus featuring a filtering behavior. The existence of elastic band gaps in periodic structures may lead to many potential applications such as sound shields, acoustic filters, transducers, refractive devices, wave guides, etc [1]. Continuing interest in such structures has seen the birth of re-entrant and chiral structures. These are unique honeycombs and foams which exhibit negative Poisson's ratio [2]. Negative Poisson's ratio foam was first developed by Lakes. Later writers have called such materials anti-rubber, auxetic materials or dilatational materials. Here we study and compare the phononic properties of both the conventional and the negative Poisson's ratio cellular structures under Hamiltonian system according to the structure-preserving properties of the symplectic algorithm.

According to the classical elasticity theory, the variation scopes of ν are from -1 to 0.5 for three dimensional (3D) isotropic materials and from -1 to 1 for two dimensional (2D) isotropic systems [3] based on thermodynamic consideration of strain energy [4]. Thus the negative Poisson's ratio effect is theoretically permissible and exists in many natural materials, such as iron pyrites, pyrolytic graphite, cancellous bone and *et al.* Because of negative Poisson's ratio effect, auxetic materials exhibit a series of fascinating properties compared with the conventional materials, such as increased shear modulus [5], increased indentation resistance [6], enhanced fracture toughness [7], better energy absorption [8, 9, 10], synclastic curvature [11, 12] and *et al.* The auxetic effect thus deserves a further research based on these fascinating advantages.

The Poisson's ratio of a material influences the transmission and reflection of stress waves [13]. It is expected that the negative Poisson's ratio foams exhibit

---

Received by the editors December 1, 2010 and, in revised form, July 4, 2011.
2000 *Mathematics Subject Classification.* 70G45,74J05.

better sound absorption trait due to the convoluted cell ribs [14]. Thus the dynamic wave dispersion and loss properties of conventional and negative Poisson's ratio cellular structures are worthwhile a further research. Experimental investigations were carried out by Chen and Lakes [15] for the dynamical behavior of conventional and negative Poisson's ratio foamed polymeric materials in torsional vibration, where dispersion of standing waves and cut-off frequencies were observed. Howell *et al* [16]demonstrated that auxetic foams exhibit better sound absorption capability than unconverted foams at all frequencies and smaller pore-size auxetic foams absorb sound more efficiently at frequencies above 630 Hz than those with larger pores. Similar conclusions were achieved by Scarpa & Tomlinson [17], who revealed that due to the improved stiffness ratios, auxetic honeycombs could offer some advantages in sound reduction applications. Foams with curved or convoluted ribs were found to possess the traits of acoustic waves dispersion and cut-off frequencies, which might lead to applications involving the absorption of sound [18].The structural-acoustic performance of the chiral core was proved to be better through comparisons with the square and hexagonal core topologies [19]. Numerical and experimental results show that auxetic structures absorb noise and vibrations more efficiently than conventional equivalents [20]. The phononic properties of a chiral cellular structure were investigated through the application of Bloch theorem where the in-plane wave propagation problem was analyzed [21]. In a word, auxetic materials show overall better energy absorption capability, such as ultrasonic, acoustic and damping, compared with conventional materials.

Based on finite element analysis and symplectic mathematics, a general method is developed here to deduce the dispersion relations for three typical cellular structures, which are hexagon, re-entrant and chiral honeycomb structures. The symplectic eigenvalue problem is introduced to analyze the phase constant surface and the dispersion relations. The characteristic of symplectic algorithm at higher frequencies calculation is emphasized in the band gap analysis. The wave beaming effect in auxetic materials guides the design of cellular structures where waves at certain frequencies do not propagate in specified directions. Numerical analysis reveals better sound insulation property of the auxetic lattice than the conventional hexagon lattice, which is the main conclusion achieved in the paper. Major difficulties in the problem considered here are for the chiral lattice structure analysis and the effective combination of the symplectic algorithm with the Bloch theorem. The findings may provide invaluable guidelines for the future application of cellular structures.

2. Geometry of the cellular structures and Poisson's ratio calculation

The behavior of a material under deformation is governed by one of the fundamental mechanical properties: the Poisson's ratio [22]. Poisson's ratio, also called the Poisson coefficient, is the negative ratio of transverse contraction strain to longitudinal extension strain in a stretched bar. Since most common materials become thinner in cross section when stretched, Poisson's ratio for them is positive. Materials with negative Poisson's ratio expand laterally when stretched and contract laterally when compressed, which is different from conventional cellular materials [7]. This unusual characteristic is achieved by forming the cells into a 're-entrant' shape which bulges inwards [23]. The three kinds of structures discussed in this paper are displayed in Fig.1.

For the conventional hexagon and re-entrant honeycombs displayed in Fig.1(a) and (b) , the cell rib is treated as a Timoshenko beam of square cross-section with