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ON ITERATIVE IMPES FORMULATION FOR TWO-PHASE FLOW WITH CAPILLARITY IN HETEROGENEOUS POROUS MEDIA

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Abstract. This work is a continuation of Kou and Sun [36] where we present an efficient improvement on the IMplicit Pressure Explicit Saturation (IMPES) method for two-phase immiscible fluid flow in porous media with different capillarity pressures. In the previous work, we present an implicit treatment of capillary pressure appearing in the pressure equation. A linear approximation of capillary function is used to couple the implicit saturation equation into the pressure equation that is solved implicitly. In this paper, we present an iterative version of this method. It is well-known that the fully implicit scheme has unconditional stability. The new method can be used for solving the coupled system of nonlinear equations arisen after the fully implicit scheme. We follow the idea of the previous work, and use the linear approximation of capillary function at the current iteration. This is different from iterative IMPES that computes capillary pressure by the saturations at the previous iteration. From this approximation, we couple the saturation equation into the pressure equation, and establish the coupling relation between the pressure and saturation. We employ the relaxation technique to control the convergence of the new method, and we give a choice of relaxation factor. The convergence theorem of our method is established under the natural conditions. Numerical examples are provided to demonstrate the performance of our approach, and the results show that our method is efficient and stable.

Key words. Two-phase flow, IMPES, Heterogeneous media, Capillary pressure.

1. Introduction

Two-phase fluid flow model in porous media is a coupled system of nonlinear time-dependent partial differential equations. We often use two different types of time discretization schemes: the fully implicit and the IMplicit-EXplicit (IMEX). The fully implicit scheme [5, 21, 23, 54, 66] implicitly treats with all terms including capillary pressure, and hence has unconditional stability and maintains the inherent coupling of two-phase flow model. This scheme results in a system of nonlinear equations. IMEX [4, 7, 31, 35, 37] generally treats the linear terms implicitly and evaluates the others explicitly, and consequently, it is conditionally stable. One advantage of IMEX is to eliminate the nonlinearity of original equations.

There are two different approaches [41, 60] used for solving the coupled system of nonlinear equations arisen after the fully implicit scheme. One is the fully coupled approach that simultaneously solves all variables and equations by a Newton-type method. Consequently, the pressures and saturations can be easily coupled at each iterative step. However, the computational cost and memory requirement will be particularly expensive, especially when the size of problems becomes large. This restricts the applications of the fully coupled approach to a certain extent. The other approach is the simulator coupling scheme that splits the entire problem into some sub-problems. These sub-problems may be solved independently by different approaches, and they are coupled by data exchanges at each iteration. Thus, the simulator coupling approach is more flexible than the fully coupled approach. If the simulator coupling approach reaches the full convergence of the iterative algorithm,

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the computed solution is the same as that of the fully coupled approach. The main advantage of the simulator coupling approach is to reduce the computational cost and memory requirement compared to the fully coupled approach [47].

To reduce the computational cost of Newton-type simultaneous methods, a number of improved approaches are presented and discussed in [3, 44, 48, 67, 72]. The reduced degree of freedom method presented in [67] uses an approximation of the Jacobian matrix in the Newton-Raphson iteration to partition the coupled system of equations into a solution of some selected primary variables plus a back substitution procedure for the solution of the other variables. A phase-based potential ordering is presented in [44] to reduce the nonlinear algebraic system arisen from the fully implicit scheme into one with only pressure dependence, and then Newton's method is applied to solve the reduced system. On the other hand, various precondition techniques are used to to increase the convergence of the Newton-type methods, for example, [9, 10, 41].

Operator splitting [1, 25, 30, 40, 46, 58] can reduce a complex time-dependent physical problem into some simpler problems based on the time-lag of dimension or physics. By operator-splitting approach, we can construct iterative operator-splitting methods [30, 32, 42], which may be used to solve the nonlinear system arisen after the fully implicit scheme.

The IMplicit Pressure Explicit Saturation (IMPES) approach is viewed as an IMEX method, which employs a splitting approach based on physics. IMPES solves the pressure equation implicitly and updates the saturation explicitly. In IMPES, we substitute the saturation constraint and Darcy's law into the sum of the two mass conservation laws to obtain the pressure equation, and explicitly treat all other variables in the pressure equation to eliminate its nonlinearity. After the pressure is obtained, we explicitly compute Darcy's velocity and two-phase saturations. As a time discretization scheme, the IMPES method is conditionally stable, and hence it must take very small time step size, especially for highly heterogeneous permeable media where the capillary pressure affects substantially on the path of fluid flow. The instability of the IMPES method [20] results from the explicit treatment of the capillary pressure and the decoupling between the pressure equation and the saturation equation.

There are numerous improved versions of IMPES for two-phase flow, for example, [16, 52, 68, 70]. Iterative IMPES is to use IMPES as an iterative scheme for full implicit systems instead of Newton iteration. This approach splits the whole equation system into a pressure and a saturation equation that are solved in the sequence as IMPES. An iterative scheme developed in [45, 50, 51] solves a pressure implicitly and an implicit saturation equation in each iteration. This implicit saturation equation is derived from the implicit capillary pressure introduced in the original saturation equation. As an iterative method, the computational cost and memory required by iterative IMPES method is smaller than the fully coupled approach at each iterative step, which is more pronounced for very large size computational problems. Iterative coupling is also popular in the simulation of single-phase and two-phase flow and reactive transport [2, 24, 27, 63–65, 71].

In this work, we pay attention to two-phase flow in heterogeneous media. Heterogeneity in capillary pressure is a computational chanllege as it may have a significant influence on flow paths [6, 8, 22, 26, 34, 38, 39, 43, 53, 56, 59, 69]. For the rocks with different permeability types, we employ the different capillary pressure functions; that is, the capillary pressure functions are discontinuous on the interfaces of rocks. In this case, the discontinuity of saturation results from the continuity of capillary