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# NEW SHOCK DETECTOR AND IMPROVED CONTROL FUNCTION FOR SHOCK-BOUNDARY LAYER INTERACTION

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**Abstract.** Standard compact scheme or upwind compact scheme have high order accuracy and high resolution, but cannot capture the shock without oscillations. In this paper, modified compact scheme is developed by using an effective shock detector to block upwinding compact scheme to cross the shock, a control function, and an adaptive scheme which uses some WENO flux near the shock. The new scheme makes the original upwinding compact scheme able to capture the shock sharper than WENO and, more important, keep high order accuracy and high resolution in the smooth area which is particularly important for shock, shock boundary layer interaction and shock acoustic interaction. The scheme is robust and has no case-related coefficients.

**Key words.** Compact Scheme, WENO, Shock-Boundary Layer Interaction, Shock Detector.

## 1. Introduction

The flow field is in general governed by the Navier-Stokes system which is a system of time dependent partial differential equations. However, for external flows, the viscosity is important largely only in the boundary layers. The main flow can still be considered as inviscid and the governing system can be dominated by the time dependent Euler equations which are hyperbolic. The difficult problem with numerical solution is the shock capturing which can be considered as a discontinuity or mathematical singularity (no classical unique solution and no bounded derivatives). In the shock area, continuity and differentiability of the governing Euler equations are lost and only the weak solution in an integration form can be obtained. The shock can be developed in some cases because the Euler equation is non-linear and hyperbolic. On the other hand, the governing Navier-Stokes system presents parabolic type behavior and is therefore dominated by viscosity or second order derivatives in the boundary layer. One expects that the equation should be solved by a high order compact scheme to get high order accuracy and high resolution. High order of accuracy is critical in resolving small length scales in flow transition and turbulence processes. However, for hyperbolic systems, the analysis already shows the existence of characteristic lines and Riemann invariants. Apparently, the upwind finite difference scheme coincides with the physics for a hyperbolic system. History has shown the great success of upwind technologies. From the point of view of shocks, it makes no sense to use high order compact schemes for shock capturing. High order compact schemes use all gird points on one grid line to calculate the derivative by solving a tri-diagonal or penta-diagonal linear system. However, the shock does not have finite derivatives and downstream quantities cannot cross shock to affect the upstream points. From the point of view of the above statements, upwind scheme is appropriate for the hyperbolic system. Many upwind or bias upwind schemes have achieved great success in capturing the shocks sharply, such as Godunov [4], Roe [15], MUSCL [19], TVD [5], ENO [6] and WENO [12, 7]. All these shock-capturing schemes are based on upwind or bias

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upwind technology, which is nice for hyperbolic systems, but is not favorable to the N-S system which presents parabolic equation behavior. The small length scale is very important in the flow transition and turbulence process and thus very sensitive to any artificial numerical dissipation. High order compact schemes [10, 20] are more appropriate for simulation of flow transition and turbulence because it is central and non-dissipative with high order accuracy and high resolution.

Unfortunately, the shock-boundary layer interaction, which is important to high speed flow, is a mixed type problem which has shock (discontinuity), boundary layer (viscosity), separation, transition, expansion fans, fully developed turbulence, and reattachment. In the case of shock-boundary layer interaction, there are elliptic (parabolic for time dependent problems) areas (separation, transition, turbulence) and hyperbolic areas (main flow, shocks, expansion fans), which makes the accurate numerical simulation extremely difficult if not impossible. We may divide the computational domain into several parts: the elliptic (parabolic for time dependent problems), hyperbolic, and mixed. The division or detection can be performed by a switch function automatically such as shock detectors which simply sets for the shock area and for the rest. The switch function may give good results for shock-boundary layer interaction, but it will have too many logical statements in the code which may slow down the computation. The switch function could also be case-related and very difficult to adjust. It would also slow down the convergence for steady problems.

A combination of compact and WENO schemes should be desirable. There are some efforts to combine WENO with standard central [9, 1] and WENO with upwinding compact (UCS) schemes [14, 22]. Their mixing function is still some kind complex and has a number of case related adjustable coefficients.

In order to overcome the drawback of the CS scheme, we need to achieve local dependency in shock regions and recover the global dependency in smooth regions. This fundamental idea will naturally lead to a combination of a local dependent scheme, e.g. WENO and global dependent compact schemes which we call "Modified Compact Scheme" (MCS).

Last year, we use WENO to improve 7th order upwinding compact scheme as we called as "modified upwinding compact scheme (MUCS)", which uses a new shock detector to find the shock location and a new control function to mix upwinding compact scheme with WENO. The mixing function is designed in following ways: the new scheme automatically becomes bias when approaching the shock, but rapidly recovers to be upwinding compact, with high order of accuracy and high resolution.

However, the mixing function must be improved for high efficiency. It is required that the mixing function must be smooth (not a switch function), keeps up-winding for shock, keeps enough dissipation before and after shock, and maintain high accuracy in the smooth region.

## 2. Modified Compact Scheme

Compact scheme is great to resolve small length scales, but cannot be used for the cases when a shock or discontinuity is involved. Our new modified compact scheme is an effort to remove the weakness by introducing WENO flux when the computation is approaching the shock.

**2.1. Effective New Shock Detector.** A very effective shock detector (Oliveira, Lu, Liu and Liu, 2009) [13] has been proposed by C. Liu. The detector has two steps. The first step is to check the ratio of the truncation errors on the coarse