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OPTIMAL $\mathcal{H}_2$ MODEL REDUCTION FOR LARGE SCALE MIMO SYSTEMS VIA TANGENTIAL INTERPOLATION

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Abstract. We consider the optimal $\mathcal{H}_2$ model reduction for large scale multi-input multi-output systems via tangential interpolation. Specifically, we prove that for general multi-input multi-output systems, the tangential interpolation-based optimality conditions and the gramian-based optimality conditions are equivalent. Based on the tangential interpolation, a fast algorithm is proposed for the optimal $\mathcal{H}_2$ model reduction. Numerical examples are presented to demonstrate the approximation accuracy and computational efficiency of the proposed algorithm.

Key Words. Optimal $\mathcal{H}_2$ model reduction, tangential interpolation, multi-input multi-output system.

1. Introduction

Model reduction is to approximate a high-order dynamic system by a low-order one. It is a fundamental tool in reducing the computational complexity of control and numerical simulation of large scale dynamical systems. It has been widely used in many applications, such as the design of very large scale integration chips, the simulation and control of microelectromechanical system devices and weather predictions. For an overview of model reduction, we refer to [1]. See also [5, 23] for more approximation problems related to control.

A commonly used method for model reduction constructs the reduced order system via tangential interpolation [11]. Interpolation-based model reduction methods produce reduced-order systems whose transfer function interpolates the transfer function of the full-order system at selected interpolation points. This class of methods is suitable for the reduction of large scale dynamical systems. However, selection of optimal interpolation points remains a challenging issue.

In this paper, we study selection of interpolation points and tangential interpolation directions that can produce the optimal $\mathcal{H}_2$ reduced-order models. The optimal $\mathcal{H}_2$ model reduction problem has been studied extensively, (see, for instance [9, 10, 12, 14, 18, 21, 22] and the references cited therein). Most researchers used first-order optimality conditions in constructing numerical algorithms. Because of the popularity of interpolation-based model reduction methods, many authors consider the problem of characterizing the optimality conditions via interpolation for the optimal $\mathcal{H}_2$ model reduction. The optimality conditions for single-input single-output (SISO) systems via interpolation were given in [12, 15]. For the multi-input

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multi-output (MIMO) systems with simple poles, the stationary conditions of the cost function were characterized via interpolation in [9]. It was shown in [8] that the stationary points of the cost function for MIMO systems (without the assumption that they have only first-order poles), can be characterized via tangential interpolation. In the literature, there is another kind of first-order optimality conditions for the optimal $\mathcal{H}_2$ model reduction problem: the gramian-based conditions. The Wilson conditions [20] and Hyland-Bernstein conditions [14] are gramian-based conditions, which use gramians of the systems.

It is important to study the connections between the two different kinds of first-order optimality conditions for better understanding existing algorithms and constructing new algorithms. In [12], the Wilson conditions and the Hyland-Bernstein conditions were proved to be equivalent. Also in [12], the interpolation-based necessary conditions for SISO systems with simple poles was proved to be equivalent to the Wilson conditions and the Hyland-Bernstein conditions. For the discrete MIMO dynamical system with simple poles, the equivalence between gramian-based conditions and interpolation-based conditions was established in [6].

Interpolation-based algorithms for the optimal $\mathcal{H}_2$ model reduction were proposed in [2, 3, 4, 12]. Algorithms were proposed in [3, 12] for SISO systems and in [2, 4] for MIMO systems. Since all these algorithms were based on the assumption that the target systems have simple poles, the ill-conditioned behavior can be expected when the target systems has multiple poles or with nearly multiple poles (cf. [8]).

In this paper, for general MIMO systems which are allowed to have multiple poles, we prove that the tangential interpolation-based optimality conditions are equivalent to the gramian-based optimality conditions. The proof of the equivalence between first-order optimality conditions for SISO systems with simple poles in [12] is based on the fact that the set of all proper rational functions with specified simple poles constitute a subspace of $\mathcal{H}_2$. However, this is not the case when the transfer functions have multiple poles. Furthermore, the transfer function of MIMO systems is a rational matrix function, which will also increase the difficulties. In this paper, we accomplish the proof by finding the relationship between the gradients of the cost function and utilizing the partial expansion of the transfer function via Jordan decomposition. Moreover, the equivalence of the first-order optimality conditions leads to the proposed tangential interpolation-based minimization algorithm. The proposed algorithm is based on solving two Sylvester equations, and then constructing the reduced order system by projection. Since the proposed algorithm does not assume that the target system has simple poles, it remains robust when the target system has multiple poles. Unlike many of existing $\mathcal{H}_2$ model reduction methods, the proposed algorithm is numerical efficient. As a result, it is suitable for very large scale dynamical systems.

The paper is organized in six sections. In Section 2, we introduce the $\mathcal{H}_2$ optimal model reduction problem and the first-order optimality conditions. In Section 3, we prove the equivalence between the two first-order optimality conditions. We propose in Section 4 a numerical algorithm based on tangential interpolation. In Section 5, numerical examples are presented to demonstrate the efficiency of the algorithm. Finally, we draw our conclusions.

2. Optimal $\mathcal{H}_2$ model reduction and first-order optimality conditions

In this section, we first describe the optimal $\mathcal{H}_2$ model reduction problem, and then introduce two first-order optimality conditions: gramian-based conditions and tangential interpolation-based conditions.