INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 7, Number 2, Pages 261–280

ON A NONLINEAR 4-POINT TERNARY AND INTERPOLATORY MULTIRESOLUTION SCHEME ELIMINATING THE GIBBS PHENOMENON

S. AMAT, K. DADOURIAN, AND J. LIANDRAT

(Communicated by Roger Temam)

Abstract. A nonlinear ternary 4-point interpolatory subdivision scheme is presented. It is based on a nonlinear perturbation of the ternary subdivision scheme studied in Hassan M.F., Ivrissimtzis I.P., Dodgson N.A. and Sabin M.A. (2002): "An interpolating 4-point ternary stationary subdivision scheme", Comput. Aided Geom. Design, **19**, 1-18. The convergence of the scheme and the regularity of the limit function are analyzed. It is shown that the Gibbs phenomenon, classical in linear schemes, is eliminated. The stability of the associated nonlinear multiresolution scheme is established. Up to our knowledge, this is the first interpolatory scheme of regularity larger than one, avoiding Gibbs oscillations and for which the stability of the associated multiresolution analysis is established. All these properties are very important for real applications.

Key Words. Nonlinear ternary subdivision scheme, regularity, nonlinear multiresolution, stability, Gibbs phenomenon, signal processing

1. Introduction

As a generalization of the binary subdivision schemes [13, 12, 10], ternary schemes have been proposed in the last years [16, 20, 27, 28, 7]. A general increasing interest for investigating higher arities has emerged since Hassan et al. [16] showed that one can achieve higher smoothness and smaller support for the so-called interpolating 4-point stationary scheme, by going from binary [12] to ternary. In [7], a non-stationary 4-point ternary interpolatory subdivision scheme has been presented. It provides the user with a tension parameter that, when increased within its range of definition, can generate at convergence C^2-continuous limit curves showing considerable variations of shape.

All these approaches deal with linear subdivision schemes and in particular the Gibbs phenomenon oscillations appear in the presence of discontinuities in the data.

On the other hand, multiresolution representations of data are useful tools in signal processing applications. Given f^L a set of data where L stands for a resolution level, a multiresolution representation of f^L is any sequence of type $\{f^0, d^0, d^1, \ldots, d^{L-1}\}$ where f^k is an approximation of f^L at resolution $k < L$ and d^k stands for the details required to recover f^{k+1} from f^k. The couple $\{f^k, d^k\}$ contains the same information as f^{k+1} and therefore the same is true for $\{f^0, d^0, d^1, \ldots, d^{L-1}\}$ and f^L.

Received by the editors September 16, 2008 and, in revised form, June 5, 2009.
2000 *Mathematics Subject Classification.* 41A05,41A10, 65D05, 65D17.
This research was supported in part by the Spanish grants MTM2007-62945 and 08662/PI/08.

Again, due to the Gibbs phenomenon, it turns out to be that the efficiency of linear multiresolution decompositions for instance for signal compression is generally limited by the presence of discontinuities.

Moreover, in signal processing applications, the multi-scale representation $(f^0, d^0, d^1, \ldots, d^{L-1})$ is usually processed obtaining $(\hat{f}^0, \hat{d}^0, \hat{d}^1, \ldots, \hat{d}^{L-1})$ that are *close to but different from* the original one. Decoding recovers the discrete set $\hat{f}^L$ from the processed representation. The stability property deals with the ability to control the error between f^L and $\hat{f}^L$ by the difference between $(f^0, d^0, d^1, \ldots, d^{L-1})$ and $(\hat{f}^0, \hat{d}^0, \hat{d}^1, \ldots, \hat{d}^{L-1})$.

Recently, various attempts to improve the classical linear subdivision schemes and their associated multiresolution algorithms have led to various nonlinear multiresolution schemes. In such frameworks, only few results for convergence and stability are available [1, 3, 5, 8, 9, 11, 21, 23, 25].

The aim of this paper is to introduce a new nonlinear ternary subdivision scheme. We successively analyze the properties of the scheme and of the associated nonlinear multiresolution transform. Convergence, regularity of the limit functions and stability of the multiresolution transform are established as well as the elimination of Gibbs oscillations in presence of discontinuities.

Up to our knowledge, this is the first interpolatory scheme of regularity larger than one, avoiding Gibbs oscillations and for which the stability of the associated multiresolution analysis is established.

The paper is organized as follows. In section 2 we introduce the basic notations and some necessary results that we use in the rest of the paper. In section 3 we present a new nonlinear ternary subdivision scheme based on the scheme studied in [16]. Writing the scheme as a perturbation of a linear scheme and establishing a contractivity property of this perturbation, we deduce the convergence of the subdivision scheme and the stability of the associated multiresolution algorithm, that due to the nonlinear nature of the scheme is not a consequence of the convergence. The elimination of the Gibbs phenomenon in presence of discontinuities is studied rigorously. Section 4 is devoted to numerical examples.

2. The basic framework

The multiresolution framework studied in this paper can be considered as a particular example of the Harten interpolatory multiresolution setting [17, 6] transposed to ternary refinement.

2.1. The Harten interpolatory multiresolution setting. One considers a set of nested bi-infinite regular grids:

$$X^j = \{x_n^j\}_{n\in\mathbb{Z}}, \quad x_n^j = n3^{-j},$$

where j is called a scale parameter. The point-value discretization (sampling) operators are defined by

$$\mathcal{D}_j : f \in C(\mathbb{R}) \mapsto f^j = (f_n^j)_{n\in\mathbb{Z}} := (f(x_n^j))_{n\in\mathbb{Z}} \in V^j,$$

where V^j is the space of real sequences and $C(\mathbb{R})$ the set of continuous functions on $\mathbb{R}$. A reconstruction operator $\mathcal{R}_j$ associated to this discretization is any right inverse of $\mathcal{D}_j$ on V^j which means that

$$(\mathcal{R}_j f^j)(x_n^j) = f_n^j = f(x_n^j).$$

The operator defined by $\mathcal{D}_{j+1}\mathcal{R}_j$ acts between the coarse level (j) and the fine level $(j+1)$ and is called a prediction operator.

Since $\mathcal{D}_{j+1}$ is the sampling operator on the grid X^{j+1} that contains the grid X^j, the prediction operator identifies with an interpolating subdivision scheme [6, 10].