INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 6, Number 2, Pages 311–334

A ROBUST AFFINE IMAGE REGISTRATION METHOD

NOPPADOL CHUMCHOB AND KE CHEN

Abstract. Image registration has many real life applications. Affine image registration is one of the commonly-used parametric models. Iterative solution methods for the underlying least squares problem suffer from convergence problems whenever good initial guesses are not available. Variational models are non-parametric deformable models that have been proposed based on least squares fitting and regularization. The fast iterative solution methods often require a reliable parametric (affine) method in a pre-registration step. In this paper we first survey and study a class of methods suitable for providing the good initial guesses for the affine model and a diffusion based variational model. It appears that these initialization methods, while useful for many cases, are not always reliable. Then we propose a regularized affine least squares approach that can overcome the convergence problems associated with existing methods. Combined with a cooling idea in a multiresolution setting, it can ensure robustness and selection of the optimal coupling parameter efficiently. Numerical examples are given to demonstrate the effectiveness of our proposed approach.

Key Words. Image registration, affine transformation, regularization, Newton method.

1. Introduction

Image registration is the process of spatially aligning two or more images of the same object taken in different times or from different viewpoints or by different imaging machineries as in multi-modality imaging. When two images are taken as input, one of them is called the reference image and is kept unchanged and used as the reference, whereas the other is called the template image and is employed to register the reference image. The goal of image registration is to determine an optimal transformation in such a way that the transformed template image becomes *similar* to the reference image as much as possible, mapping points from the template image onto the reference image. This transformation is sought from optimizing an appropriate object functional, which measures the similarity of the transformed template image to the reference image and the transformation regularity; it is the latter that we address here in a new model context.

The registration methodology can be classified into two main *physical* categories: rigid and non-rigid registration, or *mathematical* categories: linear and nonlinear registration, or *complexity* categories: parametric and non-parametric registration.

Received by the editors January 1, 2008 and, in revised form, March 22, 2008.
2000 *Mathematics Subject Classification.* 62B5, 94A08, 47A52, 90C53.
This research was supported by a Royal Thai Government Fellowship for NC. For correspondence, contact k.chen@liv.ac.uk.

On one hand, rigid registration involves a linear rigid-body transformation, consisting of rotation and translation (with only 3 unknown parameters). On the other hand, deformable registration (e.g. variational models [23]) may include nonlinear transformations (non-parametric) whose number of unknowns for a discrete image is proportional to the number of pixels!

In real-life applications, rigid registration alone cannot always provide a satisfactory result, particularly in many medical applications (e.g. one cannot ensure the patient sits in the identical position with respect to the equipment each time), while deformable registration may not be quick enough for ready use. In this paper, we are mainly concerned with affine transformation for 2 reasons: (i) it is applicable to a large class of non-rigid registration problems. Moreover, as our experience shows, an affine method is always many orders of magnitude faster than a nonlinear variational method [23] due to much less unknowns involved. See also Thévenaz *et al.*[32], Jenkinson and Smith [15], Jenkinson *et al.*[14], Modersitzki [23], Xia and Liu [36], Zhilkin and Alexander [38], Lucchese [20] and the references therein. (ii) it is widely used as a pre-registration step for sophistical non-rigid registration methods, such as elastic, fluid, and diffusion registration, by providing the good initial positions for the image to be registered (see [23] and Schmitt *et al.*[29] and the references therein).

To realize image registration, assuming the image intensities of the given images are comparable, a common approach is to minimise the Sum of Squared Differences (SSD) i.e. the least-squared function for the squared pixel-wise differences in the image intensity between the transformed template image and the reference image. Then the obtained transformation defines a transformed template image as required. For affine registration, although there are only 6 parameters, iterative methods to solve the underling nonlinear minimization can suffer from convergence problems if good initial guesses are not possible (i.e. even after we attempt to devise good initial guesses). A theoretical reason may be that image registration problem is ill-posed in the sense of Hadamard. The information provided by the reference and the least-squared model are not sufficient to ensure the existence, uniqueness, and stability of a solution [12]. This motivates us to introduce regularization into affine registration, as one would do for other ill-posed problems [1, 23, 26, 34]. The result is a refined affine registration model that can be solved by converging methods for a large class of image problems.

We remark that image registration is required whenever comparing a series of images is of interest. For example, in remote sensing applications, registration of satellite images taken over a region during different seasons or years can be used to detect environment change over time [4, 5, 27] while, in medical image processing, a vital component of applications is the registration of relevant images of a patient in order to obtain accurate information for diagnosis, monitoring disease progression, planing treatment and treatment guidance. See Maintz and Viergever [22], Hajnal *et al.* [9], and Hill *et al.* [13].

The rest of the paper is organized as follows. We introduce the affine and the diffusion image registration respectively in Sections 2 and 3, and then present four methods to improve affine registration in Section 4. A regularized affine registration (RAR) model is presented in Section 5 followed by a regularization parameter selection algorithm in Section 6. Some numerical experiments on the performance of the proposed method are presented in Section 7, followed by conclusions in Section 8.