INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 6, Number 2, Pages 284–292

IMAGE ZOOMING ALGORITHM BASED ON PARTIAL DIFFERENTIAL EQUATIONS TECHNIQUE

RAN GAO[1,2], JIN-PING SONG[1,2], AND XUE-CHENG TAI[3,4]

Abstract. Partial Differential Equations (PDEs) have become an important tool in image processing and analysis. A PDE mode for image zooming is introduced in this paper. This model exploits a higher order nonlinear partial differential equation. The resulted nonlinear equation is solved by an explicit finite difference schemes. Numerical results on real digital images are given to show effectiveness and reliability of the proposed algorithm.

Key Words. image zooming, interpolation, partial differential equations(PDEs)

1. Introduction

Due to the development of modern information technology, image processing is becoming more and more important in our life. Digital zooming is encountered in many real applications such as electronic publishing, image database, digital camera, visible wireless telephone, medical imaging and so on. In order to have better and fine images for users, images often need to be zoomed in and out or reproduced to higher resolution from lower resolution.

One common way for image zooming is interpolating the discrete source image. Interpolation is the first step of two basic re-sampling steps and turns a discrete image into a continuous function, which is necessary for various geometric transform of discrete images. There are two kinds of interpolation methods: linear and nonlinear ones. For linear methods, diverse interpolation kernels of finite size have been introduced in the literature. Approximations of the ideal interpolation kernel which is spatially unlimited are essential for these methods, see [5, 9, 17]. The simplest and most popular approximations are related to pixel replication [4], bilinear interpolation [12] and bicubic interpolation methods [7, 6]. They have been routinely implemented in commercial digital image processing softwares. Pixel replication method is a technique of nearest neighbor interpolation [13], which is simple to implement by replicating the original pixels. This method is usually susceptible to the undesirable defect of blocking effects. Bilinear and bicubic interpolation employ first-order spline and second-order spline models, respectively. By doing so, more pleasing outcome is resulted for many real digital images.

A generic zooming algorithm takes as input an RGB picture and provides as output a picture of greater size preserving the information content of the original image as much as possible. Unfortunately, the methods mentioned in the passage above, can preserve the low frequency content of the source image well, but are not equally well to enhance high frequencies in order to produce an image whose visual sharpness matches the quality of the original one. Especially, when the image is zoomed by a large factor, the zoomed image looks very often blocky [14, 15]. In

Received by the editors March 23, 2008 and, in revised form, June 17, 2008.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.

addition to the problem with sharpness, lower order methods degrade the zoomed image quality, despite the fact that they require less computation effort compared to higher order interpolation methods [2]. One of the basic concepts of the algorithms mentioned above is to interpolate images using the feature of pixels. Determination of pixel feature through these methods needs higher computational complexity, and the result is often disappointing. The method proposed in this paper tries to take into account information about discontinuities or sharp luminance variations.

In recent years, PDEs have achieved great success in the field of image processing [16, 1, 8, 18, 10, 3, 11]. Its basic principle is to use piecewise smooth surfaces to approximate images. Because of the effect of diffusion, the image which has been processed will be quite similar to the original image in edge and other places. Thus, it enables to obtain images not only have good smoothness but also preserve sharpness of the edges. Based on this characteristic, we try to proposes a PDE-based interpolation model in this paper. The basic idea of the algorithm is to introduce a fourth-order PDE to smooth the image. Our experiments show that the proposed method is better than bilinear interpolation.

The paper is organized as follows. Our PDE-based model and its numerical realization are formally introduced in Section 2. Section 3 is devoted to numerical experiments, followed by some conclusions in Section 4.

2. PDE-based Image Zooming Algorithm and Its Realization

The PDE model we shall introduce is based on a noise removal algorithm proposed in [15]. In [15], the authors proposed a fourth-order PDE to image denoising, which is to recover an image u from a noisy observation u_0. This model is referred as the LLT model. For noise removal, one needs to solve the following minimization problem:

$$(1)\ \min_u E(u),\ \text{where } E(u) = \int_\Omega (u_{xx}^2+u_{xy}^2+u_{yx}^2+u_{yy}^2)^{\frac{1}{2}} dxdy + \frac{\lambda}{2}\int_\Omega |u-u_0|^2 dxdy.$$

Assume the noise level $\sigma^2 = \int_\Omega |u-u_0|^2 dxdy$ is approximately known, then one can use a Lagrangian multiplier to solved the above constrained minimization problem. The resulted equation is a fourth order nonlinear partial differential equation. High-order PDEs are known to recover smoother surfaces better. The nonlinear PDE resulted from LLT can also preserve jump rather well. In this work, we try to use this idea for image zooming.

2.1. The PDE-based model. To formulate the problem in the continuous setting, we assume that the low resolution image $u_0(x,y), (x,y) \in \Omega_1$ is given in $\Omega_1 \subset \Omega$ and $\Omega = \Omega_1 \cup \Omega_2$. In the discrete setting, Ω_1 contains the grid points of the low resolution image pixels. From the values of u_0 in Ω_1, we want to extend them to the whole region Ω.

For this purpose, we shall try to find a function u defined in the whole region to minimize the energy functional:

$$(2)\ E(u) = \int_\Omega (u_{xx}^2 + u_{xy}^2 + u_{yx}^2 + u_{yy}^2)^{\frac{1}{2}} dxdy + \frac{\lambda}{2}\int_\Omega |u-u_0|^2 \cdot \chi_{\Omega_1}(u-u_0) dxdy.$$

Above, χ_{Ω_1} is the 'characteristic function' of Ω_1. The first term in $E(u)$ is a smoothing term and it is used to guarantee that recovered image has smooth level curves. The parameter λ is a positive weighting constant that controls contribution of fidelity term. Minimizing the continuous energy functional $E(u)$ yields a nonlinear fourth-order PDE. Thereby, we obtain a PDE-based interpolation model.