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AN EFFICIENT AND EFFECTIVE NONLINEAR SOLVER IN A PARALLEL SOFTWARE FOR LARGE SCALE PETROLEUM RESERVOIR SIMULATION

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Abstract. We study a parallel Newton-Krylov-Schwarz (NKS) based algorithm for solving large sparse systems resulting from a fully implicit discretization of partial differential equations arising from petroleum reservoir simulations. Our NKS algorithm is designed by combining an inexact Newton method with a rank-2 updated quasi-Newton method. In order to improve the computational efficiency, both DDM and SPMD parallelism strategies are adopted. The effectiveness of the overall algorithm depends heavily on the performance of the linear preconditioner, which is made of a combination of several preconditioning components including AMG, relaxed ILU, up scaling, additive Schwarz, CRP-like(constraint residual preconditioning), Watts correction, Shur complement, among others. In the construction of the CRP-like preconditioner, a restarted GMRES is used to solve the projected linear systems. We have tested this algorithm and related parallel software using data from some real applications, and presented numerical results that show that this solver is robust and scalable for large scale calculations in petroleum reservoir simulations.

Key Words. Petroleum reservoir simulation, Nonlinear solver, Preconditioning, Inexact Newton, BFGS, Krylov subspace, Parallel performance.

1. Introduction

Petroleum reservoir simulation solves the multidimensional and multiphase equations of conservation of mass in porous media, subject to appropriate initial and boundary conditions. The processes occurring in petroleum reservoirs are basically fluid flow and mass transfer. Black Oil Model [1, 2] is regarded as the fundament of reservoir simulation work, where fluids of different phases are usually considered to be at constant temperature and in thermodynamic equilibrium throughout the reservoir. There are three distinct phases, namely oil, water and gas, in this model. Flow in a porous media is governed by three kinds of equations: PDEs describing material flow between blocks which are governed by Darcy's law, a phase-constraint equation describing a saturation relationship of three different phases, capillary pressure equations describing surface tension and the curvature of the interface between the two fluids within the small pores.

In last few years, the performance of parallel petroleum reservoir simulation has been significantly improved ([3]-[10]). However, only a few papers offer their results and effects of practical reservoir problems on MPP computers with more than 32 CPUs. We have developed a parallel black-oil simulator based on a sequential code ([11]), it works well on distributed-memory machines. This simulator uses a

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fully implicit scheme to discretize the coupled PDEs. The resulting set of nonlinear equations is solved by using inexact Newton method with special choice the initial guess([12]). Efficiency, flexibility and portability are emphasized throughout processes of design and implementation. The solver package is designed and coded so that it is adapted to solving a variety of multi phase flow problems, not being limited to black-oil problems.

Newton method has attractive theoretical and practical properties. If the initial guess is close enough to the exact solution, then quadratic convergence can be obtained. In the nonlinear solver, choosing a good initial guess is one of our emphases. We use BFGS method to provide a good initial guess.

In Newton iteration the most expensive part is solving large sparse linear systems. Usually, each Newton step uses Krylov subspace method with a proper preconditioner. Numerical tests show that, comparing different Krylov subspace algorithms with their "proper" chosen preconditioners, no one algorithm is obviously better than the other ([15]). So the most important part is the choice of preconditioning strategy. Our parallel simulator uses a FGMRES method ([16])with an iterative preconditioning as a typical linear solver. The used preconditioner adopts a so-called multipurpose oblique projection correction strategy ([12]), which involves several preconditioning components such as AMG, relaxed ILU, up scaling, DDM, CRP ([17]) etc.

2. The Black Oil Model and Discretization

The three-phase flow conservation equations can be expressed as [18]

(1)

$$\begin{aligned} \nabla[T_w \nabla(P_w - \rho_w g D)] \;+\; q_w \;&=\; \frac{\partial(\phi b_w S_w)}{\partial t} \\ \nabla[T_o \nabla(P_o - \rho_o g D)] \;+\; q_o \;&=\; \frac{\partial(\phi b_o S_o)}{\partial t} \\ \nabla[T_g \nabla(P_g - \rho_g g D)] + \nabla[T_o R_s \nabla(P_o - \rho_o g D)] + q_g + R_s q_o &= \frac{\partial(\phi b_g S_g + \phi b_o S_o R_s)}{\partial t}, \end{aligned}$$

where $T_l := M_l b_l$ is the transmissibility of phase-l $(l = w, o, g)$, $b_l := f_1(P_o)$ is the reciprocal of formation volume factor, D is the vertical depth, $R_s := f_2(P_o)$ is the gas-oil ratio, and $\phi := f_3(P_o)$ is the rock porosity. As a factor of T_l, the mobility $M_l := \dfrac{K f_4(S_w, S_g)}{\mu_l}$ gives a relationship between the flow rate $\vec{v}_l$ and the pressure gradient ∇P_l in each phase through Darcy's Law

$$\vec{v}_l = -M_l \nabla(P_l - \rho_l g D) \ .$$

As an empirical fact, the capillary pressure is a unique function of saturation which provides a relationship between different phase pressures

$$P_w = P_o - P_{cow}(S_w) \ , \ P_g = P_o + P_{cgo}(S_g) \ .$$

As a result, the three unknowns of the above PDEs are oil-phase pressure (P_o), water-phase and gas-phase saturation (S_w, S_g). More details of the variables and their physical properties can be found in many literatures, e.g. ([2]). This model is being used in the commercial reservoir simulation software packages such as VIP ([7]), ECLIPSE ([8]), IPARS([9]) and Simbest-II ([11]). The model represents mathematically a class of important industrial problems rather than simply being an idealized model for benchmark tests and uses realistic saturation coefficients, permeability, and transmissibility which are in-situ field data collected over a long period of time.