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A UNIFIED PARALLEL DEA MODEL AND EFFICIENCY MODELING OF MULTI-ACTIVITY AND/OR NON-HOMOGENEOUS ACTIVITY

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Abstract. Data envelopment analysis(DEA), as originally proposed, is a methodology for evaluating the relative efficiencies of peer decision making units (DMUs) under some general assumptions. DEA models with non-homogeneous DMUs and multi-activity structures are two different subjects referring to relaxing various assumptions. In this paper, we show that these two formulations are both derived by embedding the corresponding process into a general parallel DEA model. Furthermore, following the parallel DEA framework, general DEA models for multi-activity and non-homogeneity are proposed, which are able to handle many situations where different aspects of non-homogeneity or multi-activities exist. This study provides important insights into the existing DEA models for non-homogeneity and multi-activity.

Key words. Data envelopment analysis, parallel model, multi-activity, non-homogeneous.

1. Introduction

Data Envelopment Analysis (DEA) has been a standard methodology for evaluating the relative performance of Decision Making Units (DMUs) since the paper of Charnes et al. [4]. Some underlying assumptions are common to traditional DEA models. DMUs are treated as black boxes since the internal structures of DMUs are ignored in traditional DEA models. Furthermore, all DMUs are considered to be homogeneous, that is they all utilize the same types of inputs to produce the same types of outputs. In the last four decades, thousands of articles and extensive work have appeared to relax the above assumptions, see [11, 29].

In some contexts, the knowledge of the internal structure of DMU can give further insights for the performance evaluation. Extensive studies have been done to model internal structures and networks of the operation, in e.g., [5, 12, 21, 23, 25, 30] and so on. Comprehensive discussions on network DEA have been showed in the handbook of Cook and Zhu [15]. A basic type of network structure is parallel system where a production system is consisted of several subsystems. In the case that a production system with parallel production units, there are currently two fundamental researches: YMK model[32] and Kao's parallel DEA model[22], and we will first show that they are both special cases of a general parallel DEA model.

Multi-activity problem is a special case concerning the internal structure of DMUs, where there exist shared inputs and outputs allocated to various activities. Furthermore, the allocation of resources remains to be determined. This problem was first studied by Beasley[1]. Molinero[28] extend the problem by concentrating on the dual of Beasley's model. Although the problem was first known as the joint determination of efficiency within a DEA context, it is often referred to as a multi-activity or multi-component model now. Subsequently, Cook et al.[10], Jahanshahloo et al. [19] and Tsai and Mar Molinero[31] have revised the model. Many authors have introduced variants of multi-activity models. Castelli et al. [2] have

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reviewed some extensions of multi-activity models, which include considering weight restrictions (Beasley[1]), variable returns to scale (Tsai and Molinero[31]), different weights on shared inputs (Cook et al.[10]), additive objective function (Cook and Hababou[8]) and various forms of inputs/outputs (e.g., Cook and Green[9]; Jahanshahloo et al.[19, 20]). Moreover, the production process of a DMU may contain several stages in which some outputs produced by a former stage are used as inputs to a later stage of production. Färe and Grosskopf[16, 17]thus proposed a network DEA model for measuring efficiency for those DMUs with multiple production stages. Subsequently, Chen [6], Chen et al.[7], Yu and Lin[33], Yu and Fan[34], and Wang et al.[35] studied multi-activity network DEA models which incorporated multiple activities and multiple processes into a unified framework. In this paper we show that multi-activity DEA models can be derived from the general parallel DEA model.

Besides the black-box assumption, in the traditional DEA models, all DMUs are known as homogeneous in the sense that each has the same type of inputs and outputs. However, in some applications such as plants and universities, this assumption may be violated. The situation that the inputs and outputs of a set of DMUs or their input-to-output relations are not exactly the same is known as non-homogeneity. The DEA model with non-homogeneous DMUs is first studied by Molinero[27] with a specific university example, and then a systematic study has been presented by Cook et al. [13, 14]. Following the work of Cook et al. [13, 14], a few extensions around the non-homogeneity phenomenon have been carried out. Imanirad et al. [18] introduced a methodology to allow for efficiency measurement in situations where some DMUs have different input-to-output relations. And in the latest paper, Liang et al.[24] examine non-homogeneity on the input side. In the current paper, by looking inside the non-homogeneity phenomenon, we will show that Cook's model can also be transformed into the form of the general parallel DEA model.

Hence following the parallel DEA framework, general DEA models for multi-activity and non-homogeneity are proposed, which are able to handle many situations where different aspects of non-homogeneity or multi-activities exist.

The paper is organized as follows. After introduction, two classic parallel DEA models are introduced and a general parallel DEA model is proposed in Section 2. Multi-activity models are presented in Section 3, in addition, how to apply the parallel DEA model is explained. Section 4 describes the specific situations of multi-activity models which exist in the literatures. Following the formulation of the general parallel DEA model, a general DEA model with non-homogeneous DMUs is proposed in Section 5. Furthermore, in Section 6 we apply the general models into specific situations. Discussions around the general model and some future directions are presented in Section 7.

2. DEA models for systems with parallel structure

A basic type of internal structure is parallel system. For a system composed of several processes connected in parallel, there are at least two fundamental researches. One is YMK model proposed by Yang et al.[32] in measuring the efficiencies of the production system with independent subsystems, the other is proposed by Kao[22] dealing with the case where all parallel subunits consume the same set of inputs to produce the same set of outputs. We think that the key difference between the YMK model and Kao's parallel DEA model lies in that the corresponding inputs and outputs across subunits are perceived to be (equally) compensable or not