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EFFICIENT FINITE DIFFERENCE METHODS FOR ACOUSTIC SCATTERING FROM CIRCULAR CYLINDRICAL OBSTACLE

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Abstract. We consider efficient finite difference methods for solving the three-dimensional (3D) acoustic scattering by an impenetrable circular cylindrical obstacle. By using the separation of variable and other techniques, we first transform the 3D problem into a series of one-dimensional (1D) problems in this paper, and then construct some efficient and accuracy finite difference methods to solve these 1D problems instead of the 3D one. There are mainly two advantages for these methods: one is that they are pollution free for the problem to be considered in this paper; and the other is that the linear systems generated from these schemes have tri-diagonal structures. These features lead to easy implementation and much less computational cost. Numerical examples are presented to verify the efficiency and accuracy of the numerical methods, even with the wave number greater than 100.

Key words. Helmholtz equation, circular cylindrical coordinate, finite difference method, pollution free, 3D ocean waveguide.

1. Introduction

In this work, we investigate the 3D acoustic scattering by an impenetrable circular cylindrical obstacle in a 3D shallow ocean waveguide. The shallow ocean waveguide considered here is in an open domain of homogeneous medium between two horizontal boundaries, and the sea surface is pressure release and the sea floor is rigid. This problem can be formulated by the Helmholtz equation with appropriate boundary conditions.

In the past decades, a large number of analytical methods have been developed to deal with the solution of acoustic propagation problems in ocean environments in (see [1, 2, 3, 4, 5, 6, 8, 11, 17, 32] and references therein). Contrary to the analytical methods, we are concerned with efficient and accurate numerical methods for the scattering problem. Many classical numerical methods have been used to solve this problem, such as, finite difference methods [12, 18, 29, 30, 34], boundary integral equation methods [9], finite element methods [14, 15, 20, 21, 26] and spectral methods [24]. In [9], the boundary integral equation method is used to compute the scattered field from the 3D bathymetry in an ocean waveguide. They solve a sequence of 1D integral equations instead of a very large two-dimensional (2D) one because of the azimuthal symmetry. This method is suitable for low-frequency, compact deformation scattering problems where the required number of discrete range steps and azimuthal components are not large. Finite element method is popular to simulate the acoustic scattering (see [15] and references therein). However, the feasible finite element method appears only for low and intermediate frequencies. Pan et al. [20] considered a coupled finite element and DtN mapping method to solve the acoustic scattering problem with an infinite long rectangle cylinder in an

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oceanic waveguide. The results show that the proposed method is valid and very fast. However, the maximum value of kL tested there is 6.5, with k and L being the wave number and the width of the infinite long rectangle cylinder, respectively. This means that the wave number is quite small. A super-spectral finite element method was developed for the acoustical wave propagation in nonuniform waveguides in [21]. This method is based on a finite-element approach using a mixture of high order shape functions and wave solutions. The computational cost has been drastically reduced.

Although lots of work have been done, an indisputable fact is that huge computational cost is required for the higher dimensional problem. The solution of this problem is highly oscillatory with large wave numbers and the "pollution effect" (see [15]) exists in almost all of these methods.

On the other hand, the finite difference method is also a popular and powerful computational technique for simulating the wave propagation modeling for its easy implementation and computational efficiency [35]. Furthermore, it can be easily extended to the 3D case. Recently, a novel kind of finite difference methods is proposed by Wang et al. to solve the 2D and 3D Helmholtz equations with large wave numbers in the polar and spherical coordinates (see [30]). The main idea of the method is to use the separation of variables and variable transformation to reduce the higher dimensional Helmholtz equation on a special domain into a sequence of 1D equations, and then construct pollution free schemes for approximating the 1D problems. The idea is extend to solving the singularly perturbed equations in [12].

In this paper, we will extend the method proposed in [30] and construct a more accurate finite difference scheme to solve 3D the waveguide problem in the circular cylindrical coordinate. Including applying the algorithms proposed in [30], we also construct a more accurate scheme to simulate the problem. The motivations are as follows: First, due to the circular cylindrical obstacles geometry, via the circular cylindrical coordinate transformation and separation of variables, we can transform the 3D problem into a series of 1D problems similar to [30]. Second, in realistic environments, usually, the magnitude of ocean waveguide depth is less than $10^3 m$, the frequencies are no more than $1000hz$, and the sound velocity is commonly $1500m/s$ in ocean waveguide, indicating that the non-dimensional wave number k is less than 1. However, in the numerical experiment, by scale shift, the magnitude of ocean waveguide depth is less than 10, and correspondingly, the wave number k is probably 100. This will result a huge linear system if solving in 3D directly because of the "pollution effect". Finally, to the best of our knowledge, most of the works are focused on the problems in 2D (range and depth) or 3D without considering azimuth.

The rest of the paper is organized as follows. In Section 2 we transform the 3D problem to a series of 1D problems and introduce the spectral normal mode solution in the circular cylindrical coordinate. Then, we construct the new finite difference schemes to solve the 1D problem in Section 3. In Section 4 we examine the performance of the scheme by testing a series of numerical experiments. Conclusions are presented in Section 5.

2. Ocean waveguide in 3D

We consider the waveguide in an open domain $\Omega \subset \mathbb{R}^3$ full of homogeneous medium between the two horizontal boundaries $z = 0$ (called 'top') and $z = H$ (called 'bottom'), where the sea surface $z = 0$ is pressure release (such as air), the sea floor $z = H$ is rigid (such as rock), and a sound soft of the immersed circular