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An h-Adaptive RKDG Method for Two-Dimensional Detonation Wave Simulations

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Abstract. An h-adaptive Runge-Kutta discontinuous Galerkin (RKDG) method with a positivity-preserving technique to simulate classical two-dimensional detonation waves is developed. The KXRCF troubled-cell indicator is used to detect the troubled cells with possible discontinuities or high gradients. At each time-level, an adaptive mesh is generated by refining troubled cells and coarsening others. In order to avoid the situations where detonation front moves too fast and there are not enough cells to describe detonation front before it leaves, a recursive multi-level mesh refinement technique is designed. The numerical results show that for smooth solutions, this h-adaptive method does not degrade the optimal convergence order of the nonadaptive method and outperforms it in terms of computational storage for shocked flows.

AMS subject classifications: 65M60, 65M99, 35L65

Key words: Runge-Kutta discontinuous Galerkin method, troubled-cell indicator, h-adaptive method, detonation wave.

1. Introduction

The proper understanding of the detonation waves propagation plays an important role in protecting human lives and avoiding property damages. To study detonation phenomena, various numerical methods have been employed, including second order Godunov scheme [1, 27], extended space-time CE/SE method [35], unsplit scheme [23], non-MUSCL-type TVD scheme [31], classical weighted essentially non-oscillatory (WENO) scheme [11, 32], optimal WENO-Z scheme [14, 15], hybrid central-WENO scheme [13], Runge-Kutta discontinuous Galerkin (RKDG) method [3,33,34] and adaptive finite volume methods [17] .

It is known that discontinuous Galerkin (DG) finite element method [4,6–8] can handle complicated geometry and h-p adaptation, and can be efficiently implemented on parallel computers. As the result, it finds various applications [5, 36, 41]. One can relatively easily

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choose an h-adaptive strategy to develop a DG method since the mesh refinement or coarsening does not use the continuity through cell interfaces. Moreover, compared to finite volume or finite difference adaptive mesh refinement schemes, DG methods are extremely convenient and computationally efficient in data prolongation and data projection at different mesh levels and in preservation of cell averages for conservation. On the other hand, they are more flexible than continuous finite element methods if there are hanging nodes in adaptive meshes. For more details on h-adaptive discontinuous Galerkin methods we refer the reader to [10, 16, 26, 29, 38–40].

Let us note that although there is an h-adaptive RKDG method for one-dimensional detonation waves [37], all realistic detonation phenomena are essentially three-dimensional. Nevertheless, many important detonation structures can be detected during two-dimensional simulations — e.g. regular or irregular cellular structures and triple-shock Mach intersection in shock interface. Therefore, here we consider an h-adaptive RKDG method with troubled-cell indicators for various classical two-dimensional detonation waves. Comparing this method with fixed-mesh RKDG method, we note significant advantage with respect to computational storage and solution quality. In particular, the multi-level mesh refinement technique guaranties that the finest mesh is generated in the neighbourhoods of discontinuities. In addition, the KXRCF troubled-cell indicator is used to detect the discontinuity regions and a high order positivity-preserving technique [33] is adopted to avoid the potential occurrence of negative pressure, density or mass fraction.

This paper is organised as follows. In Section 2, we introduce the RKDG method. An h-adaptive RKDG method for the two-dimensional reactive Euler equations and its implementation are considered in Section 3. The performance of this adaptive technique is demonstrated on several classical examples in Section 4 and our concluding remarks are in Section 5.

2. Review

We consider the system of two-dimensional conservation laws with a source term

$$\mathbf{w}_t + \mathbf{f}(\mathbf{w})_x + \mathbf{g}(\mathbf{w})_y = \mathbf{s}(\mathbf{w}), \quad t > 0, \quad (x, y) \in \Omega, \tag{2.1}$$

where $\mathbf{w}$ is the conserved variable vector, $\mathbf{s}(\mathbf{w})$ the source vector and $\mathbf{f}(\mathbf{w})$ and $\mathbf{g}(\mathbf{w})$ are the flux vectors. The two-dimensional governing equations for modeling the ideal gaseous detonation has the form

$$\begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \\ \rho f_1 \end{pmatrix}_t + \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ (E+p)u \\ \rho f_1 u \end{pmatrix}_x + \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ (E+p)v \\ \rho f_1 v \end{pmatrix}_y = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega \end{pmatrix}, \tag{2.2}$$

where ρ is the density, (u, v) the velocity vector, P the pressure, and $f_1 \in [0, 1]$ the reactant mass fraction. Let us also note that the total energy E of the system can be expressed in