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The Contraction Coefficient of a Free-Surface Flow Under Gravity Entering a Region Beneath a Semi-Infinite Plane

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Abstract. Borda's mouthpiece consists of a long straight tube projecting into a large vessel, where fluid enters the tube in a free surface flow that tends to become uniform far downstream in the tube. A two-dimensional approximation to this flow under gravity in the upper part of the tube leads to an evaluation of the contraction coefficient, the ratio of the constant depth of the uniform flow to the width of the tube. The analysis also applies to flow under gravity past a sluice gate, if the semi-infinite wall above the channel is rotated to the vertical. The contraction coefficient depends upon the Froude number F, and is generally less than the zero gravity value of $1/2$ that is approached as $F \to \infty$.

AMS subject classifications: 65E05, 76B07, 76M15

Key words: Borda's mouthpiece, free-surface flow, boundary integral equation, contraction coefficient.

1. Introduction

Borda's mouthpiece consists of a long straight tube projecting into a large vessel, as illustrated in Fig. 1. The fluid in the vessel flows into the tube with a free surface detached from the wall of the tube, and eventually in a uniform stream far from its entry into the tube. When gravity is neglected, the flow in the tube is symmetric about the centre of the tube, and the ratio between the width of the uniform stream and the diameter of the tube (called the *contraction coefficient*) is known to be $1/2$ [1]. The symmetric flow assumption in Borda's mouthpiece is not valid when gravity is included, but in this paper we ignore the flow into the lower part of the tube (indeed, the flow in the entire shaded part of

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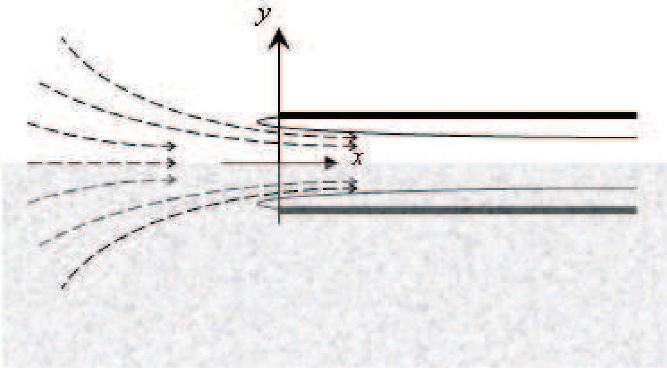

Figure 1: Borda's mouthpiece flow.

Fig. 1) and consider a two-dimensional approximation of the flow into its upper part. The contraction coefficient of the assumed two-dimensional flow under gravity is calculated in this paper.

Our analysis also applies to flow under gravity past a sluice gate, if the semi-infinite horizontal wall above the channel in the corresponding two-dimensional configuration shown in Fig. 2(a) is rotated to the vertical. A numerical solution has been found for the free surface flow under a sluice gate from deep water, and the modified flow due to the channel terminating in a waterfall was also considered [2, 3]. Most sluice gate flows discussed elsewhere are for fluid of finite depth upstream [4–6] — cf. also [7–9] more recently. The solutions are characterised by uniform and supercritical flow far downstream, and supercritical or subcritical flow far upstream. A wave train occurs when the upstream flow is subcritical. Binder and Vanden Broeck [10] considered possible multiple disturbances at the bottom of the channel or the free surface, such as due to a submerged obstacle or a pressure distribution or a sluice gate. They obtained solutions in which the radiation condition is satisfied — i.e. waves are formed near the gate and disappear, so the flow tends to be uniform far upstream. However, all solutions involve uniform and supercritical flow far downstream. We found this characteristic behaviour in our numerical solution for the flow from infinitely deep water [2] — but also a back flow near the edge of the gate and a stagnation point for small Froude number, and free surface separation from the vertical wall at an angle $2\pi/3$ or $-5\pi/6$ to the vertical axis [3]. This limiting case agrees with Vanden-Broeck & Tuck [11], who considered a free surface flow locally near a vertical wall — cf. also Dagan & Tulin [12] for analysis of the flow near a stagnation point. A solution with a stagnation point was also obtained for an inclined wall until the wall makes the angle $\pi/3$ to the horizontal and the stream leaves the wall horizontally [14], when the free-surface flow leaves the boundary smoothly without any stagnation point. Mc Cue and Forbes [13] solved free-surface flows emerging from a semi-infinite plate with constant vorticity, a stagnation point occurs at the end of the plate and the free surface leaves the plate at detachment angle $2\pi/3$ for small Froude number.

In this paper, we use a boundary element method to solve an integral equation involv-