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Modulus-Based Multisplitting Iteration Methods for a Class of Nonlinear Complementarity Problems

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Abstract. Modulus-based multisplitting iterative methods for large sparse nonlinear complementarity problems are developed. The approach is based on a reformulation of nonlinear complimentarily problems as implicit fixed-point equations and includes Jacobi, Gauss-Seidel and SOR iteration methods. For systems with positive definite matrices the convergence of the methods is proved. The methods are suitable for implementation on multiprocessor systems and numerical experiments confirm their high efficiency.

AMS subject classifications: 90C33, 65H10, 65K05

Key words: Nonlinear complementarity problem, modulus-based multisplitting methods.

1. Introduction

We consider the nonlinear complementarity problem (NCP), which consists in finding a vector $u \in \mathbb{R}^n$ such that

$$u \geq 0, \quad v = F(u) \geq 0, \quad u^T v = 0, \tag{1.1}$$

where $F(u) = Au + \Phi(u) + q$, and $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ is a large sparse real matrix, $q \in \mathbb{R}^n$ and $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ is a diagonal differentiable mapping such the i-th component Φ_i of Φ is a function of i-th variable u_i only — i.e.

$$\Phi_i = \Phi_i(u_i), \quad i = 1, 2, \cdots, n.$$

If Φ is a linear function, the problem (1.1) reduces to a linear complementarity problem (LCP).

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We call (1.1) the nonlinear complementarity problem with nonlinear source term. It has numerous applications in engineering, including discrete simulation of the Bratu obstacle problem and free boundary problems with nonlinear source terms [12, 19, 20]. Various numerical methods have been used to find approximate solution of this problem — e.g. linearised projected relaxation methods [13], multilevel methods [25], domain decomposition methods [1, 16], semi-smooth Newton method [21] and so on. Many of these methods employ the solutions of linear complementarity subproblems — cf. Refs. [4, 5, 7, 8].

The matrix splitting method is widely used in solving the systems of linear equations. Extending it to LCP, Bai [6] developed a unified approach to the construction of modulus-based matrix splitting iteration methods. The method proved to be very efficient and attracted considerable attention [15, 17, 18, 24, 26]. Applications to nonlinear complementarity problems have been also considered [22, 23]. On the other hand, the development of high-speed multiprocessor systems stimulated the study of multisplitting iteration methods. Frommer and Mayer [14] proposed multisplitting methods for linear systems that are suitable for parallel computing. Afterwards, a series of nonlinear multisplitting relaxation methods for NCP have been sudied [2, 3, 9].

Recently, new modulus-based synchronous multisplitting iteration methods for LCP, suitable for high-speed parallel multiprocessor systems, have been considered [10]. These modulus-based matrix splitting iteration methods contain, among others, the multisplitting relaxation methods (Jacobi, Gauss-Seidel), successive overrelaxation, and accelerated overrelaxation of the modulus type. In the present work, we would like to extend them to NCP.

This paper is organised as follows. In Section 2, the modulus-based multisplitting iteration methods are introduced. The convergence of these methods for positive definite system matrices is considered in Section 3. Section 4 contains numerical examples.

2. Modulus-Based Multisplitting Iteration Methods

Let $P \in \mathbb{R}^{n\times n}$ be a nonsingular matrix. On the space $\mathbb{R}^n$ it induces the norm $\|x\|_{P,2} := \|Px\|_2$, where $\|\cdot\|_2$ refers to the usual Euclidean norm. Note that for any $X \in \mathbb{R}^{n\times n}$ the corresponding matrix norm has the property $\|X\|_{P,2} = \|PXP^{-1}\|_2$— cf. Ref. [6].

Let $A, M \in \mathbb{R}^{n\times n}$. If M is a non-singular matrix, then the representation $A = M - N$ is called the splitting of the matrix A. The following theorem shows that the problem (1.1) can be transformed into an equivalent system of fixed-point equations.

Theorem 2.1 (cf. Xie *et al.* [23])**.** *Let $\Omega \in \mathbb{R}^{n\times n}$ be a nonnegative diagonal matrix, $A = M - N$ the splitting of $A \in \mathbb{R}^{n\times n}$ and h a positive number. Then:*

(i) If (u, v) is a solution of (1.1), then $x = (u - \Omega^{-1}v)/h$ satisfies the implicit fixed-point equation

$$(\Omega + M)x = Nx + (\Omega - A)|x| - 2h^{-1}\left(q + \Phi\left(2^{-1}h(|x| + x)\right)\right). \tag{2.1}$$

(ii) If x satisfies the implicit fixed-point equation (2.1), then

$$u = 2^{-1}h(|x| + x), \quad v = 2^{-1}h\Omega(|x| - x),$$