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Simulation of a Soap Film Möbius Strip Transformation

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Abstract. If the closed wire frame of a soap film having the shape of a Möbius strip is pulled apart and gradually deformed into a planar circle, the soap film transforms into a two-sided orientable surface. In the presence of a finite-time twist singularity, which changes the linking number of the film's Plateau border and the centreline of the wire, the topological transformation involves the collapse of the film toward the wire. In contrast to experimental studies of this process reported elsewhere, we use a numerical approach based on the immersed boundary method, which treats the soap film as a massless membrane in a Navier-Stokes fluid. In addition to known effects, we discover vibrating motions of the film arising after the topological change is completed, similar to the vibration of a circular membrane.

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Key words: Möbius strip, minimal surface, twist singularity, topological change, Navier-Stokes equation, immersed boundary method.

1. Introduction

Numerical studies of a soap film Möbius strip are conducted within the fluid dynamics framework. The Möbius strip has become well known since its discovery by Möbius and Listing independently in 1858 [40]. The topological change of a minimal Möbius strip occurs when the doubly looped wire frame supporting the Möbius strip is pulled apart and gradually deformed into a planar circle.

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For a smooth surface $\Sigma(t)$, the first variation of area formula is

$$\left.\frac{d}{dt}\right|_{t=0} \mathrm{Area}\big(\Sigma(t)\big) = 2\int_{\Sigma(0)} H\,\mathrm{dA}\,, \tag{1.1}$$

where H is the mean curvature [36]. If $H = 0$ the corresponding surface Σ can therefore have a relative minimal area, so zero mean curvature is commonly used for determination of minimal surfaces. The study of minimal surfaces with a closed boundary was initiated by Euler, and Courant investigated minimal surfaces and conducted various experiments with soap films spanning different wire frames [11]. Courant raised several questions concerning the existence and uniqueness of minimal surfaces for fixed wire frames, and the dependence of minimal surfaces on the prescribed motion of the wire frame [11].

The problem of the existence of a minimal surface with prescribed boundary conditions, known as Plateau's problem [41], has been solved independently by Douglas [13,14] and Radó [42,43]. Successive theoretical studies were mainly focused on more general boundary conditions, the extension of minimal surface theory in higher dimensions, and the classification of embedded minimal and periodic minimal surfaces [9,36]. In addition, numerical methods have been used to find approximate minimal surfaces — e.g. finite difference [12], finite element [4,10,16,22], level set [6,8,35] and boundary element methods [20], and also a high-order convergent numerical scheme [47].

Although Plateau's problem has been studied in the framework of elliptic variational problems [46], for non-orientable surfaces and for surfaces with multiple junctions [21], the uniqueness of minimal surfaces and their dependence on moving boundaries have received much less attention [32,33]. On the other hand, the continuous dependence of minimal surfaces on the prescribed motions of their boundaries is closely connected to the shapes of a soap films supported by deforming wire frames. Topological changes of the soap-films arising from the wire frame deformations have also received considerable attention, and vigorously studied using drops of viscous or inviscid fluids [17,34], bubbles [2,15,24,31], a foam structure [5,25,48], and a soap-film breakup [7,18,44]. However, the changes of minimal surfaces have been investigated for only a few simple surfaces — viz. from a catenoid to two disks when the distance between two supporting rings grows, and from a helicoid to a ribbon-shaped surface when the twist angle of two supporting helices increases [3].

The Möbius strip is a minimal surface that can be made physically. Thus by dipping an almost doubly-looped wire frame into a liquid soap, and then lifting out the wire frame and breaking the appropriate soap surfaces, the soap film remaining forms a Möbius strip — cf. Fig. 1. If the closed wire frame is gradually deformed into a planar circle, it has been found that the soap film spanning the wire can be transformed into a two-sided orientable surface. The topological change of such soap-film Möbius strips has been investigated by Goldstein *et al.* [19]. It was discovered that this process involves the collapse of the soap-film toward the wire frame and a finite-time twist singularity changing the linking number of the film's Plateau border and the centreline of the wire. The twist singularity is defined as the occurrence of a singular point on the surface accompanied by a discontinuous linking number. If the twist singularity occurs, the surface has at least a one non-regular point, and