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Variational Principles for Vibrating Carbon Nanotubes Conveying Fluid, Based on the Nonlocal Beam Model

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Abstract. Variational principles are derived in order to facilitate the investigation of the vibrations and stability of single and double-walled carbon nanotubes conveying a fluid, from a linear time-dependent partial differential equation governing their displacements. The nonlocal elastic theory of Euler-Bernoulli beams takes small-scale effects into account. Hamilton's principle is obtained for double-walled nano-tubes conveying a fluid. The natural and geometric boundary conditions identified are seen to be coupled and time-dependent due to nonlocal effects.

AMS subject classifications: 58E30, 74H45, 82D80

Key words: Variational principles, carbon nanotube, fluid flow, nonlocal beam, vibration.

1. Introduction

The many uses of carbon nanotubes include their application as nanopipes to convey liquids. Various aspects of the physics of the flow through a nanotube acting as a pipe have been considered [19, 25, 42, 49], together with relevant computational models [26, 27, 33] — cf. also their assessment by Wang & Ni [45]. The transportation of fluids through nanotubes can lead to vibrations and dynamic instability. Understanding this phenomenon is of practical and theoretical importance, and the instability has been analysed in a number of studies [18, 37, 46, 47, 51, 54, 55]. A review on the dynamics of liquids in carbon nanotubes has been given by Mattia & Gogotsi [34].

Continuum models have successfully described the mechanical behaviour of carbon nanotubes in a number of studies [8]. Thus the prediction of the dynamic behaviour of nanotubes conveying fluid have involved Euler-Bernoulli and Timoshenko beam and shell models rather than some molecular dynamics description. Recent studies on the vibration and stability of fluid-conveying nanotubes using continuum models are based on classical local elastic theory, where the stress is related to the strain at any point — e.g. see Refs. [17, 26, 29, 33, 36, 42, 45, 46, 49-53]. However, as such they neglect small-scale effects, and the accuracy of a continuum model can be improved by using *nonlocal* constitutive equations.

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Nonlocal theory, where the stress at any given point is assumed to depend not only on the strain at that point but also on the strain at all points in the structure, enables small-scale effects to be taken into account.

There are two different nonlocal theories that have been used to describe nanotube mechanics — viz. the nonlocal theory based on Eringen's work [15, 16], and the strain gradient theory [7]. Indeed, both theories have been employed to study the dynamics of fluid-conveying nanotubes. Wang *et al.* [44], Wang [41] and Rafiei *et al.* [36] used a nonlocal Euler-Bernoulli model. Kiani [28] employed a nonlocal Rayleigh model based on Eringen's theory, and coupled stress theory was used by both Wang [41] and Zeighampour & Beni [56]. One phenomenon affecting the accuracy of such models is the ovalisation of cylindrical shells subject to nonlinear vibrations [9]. Ovalisation has been observed in nanotubes subjected to loads in the post-buckling range [17, 38], but it is negligible for shells operating in the linear range [40]. Simply supported fluid-conveying nanotubes have mostly been studied, when analytical solutions based on trigonometric functions may be found. However, approximate and numerical methods are required when there are more complicated boundary conditions, such as free ends — e.g. for cantilever nanopipes [55]. Natural boundary conditions can be identified in variational formulations, which also provide physical insight by including expressions for kinetic and potential energies. Previously, variational principles were derived for shear deformable nanobeams [9], multi-walled nanotubes undergoing buckling [1, 2], and vibrations [3-5,28]. To describe the mechanical behaviour, a nonlocal Euler-Bernoulli beam model [1, 3, 4], a nonlocal Timoshenko beam model [2, 10, 29], and a strain-gradient shell model [5] have been used.

The objective of this article is to derive variational principles and identify the natural boundary conditions for single-walled and double-walled carbon nanotubes conveying fluid and undergoing vibrations, via a nonlocal Euler-Bernoulli beam model. Thus previous variational results on the buckling and vibration of nanotubes are extended to consider the flow through the nanotube, which may cause dynamic instability. Preliminary results were presented earlier at a conference [6]. Variational formulations for the single and double-walled nanotubes are derived from the corresponding differential equation formulations. The semi-inverse variational method developed by He [20,21], previously applied to several problems of mathematical physics [21-23, 31], is adopted in the case of the double-walled nanotube. Hamilton's Principle is a quite important feature of the variational formulation, as is also the coupling of the time-dependent natural and geometric boundary conditions in the presence of a small scale parameter. The single-walled nanotube is considered in Section 2, and double-walled nanotubes in Section 3. The relevant boundary conditions for each case are discussed in Section 4, and concluding remarks are made in Section 5.

2. Single-Walled Nanotube

2.1. Governing equations

A single-walled carbon nanotube that carries fluid is modelled as a circular Euler-Bernoulli beam of radius R and length L, with the x-coordinate in the longitudinal direction