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Ruin Probability in a Generalised Risk Process under Rates of Interest with Homogenous Markov Chains

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Abstract. This article explores recursive and integral equations for ruin probabilities of generalised risk processes, under rates of interest with homogenous Markov chain claims and homogenous Markov chain premiums. We assume that claim and premium take a countable number of non-negative values. Generalised Lundberg inequalities for the ruin probabilities of these processes are derived via a recursive technique. Recursive equations for finite time ruin probabilities and an integral equation for the ultimate ruin probability are presented, from which corresponding probability inequalities and upper bounds are obtained. An illustrative numerical example is discussed.

AMS subject classifications: 62P05, 60G40, 12E05

Key words: Integral equation, recursive equation, ruin probability, homogeneous Markov chain.

1. Introduction

Ruin probabilities in discrete time models have been considered by many authors. Teugels & Sundt [8, 9] studied the effects of a constant rate on the ruin probability under the compound Poisson risk model. Yang [11] established both exponential and non-exponential upper bounds for ruin probabilities in a risk model with constant interest force and independent premiums and claims. Xu & Wang [10] investigated a discrete-time risk model with constant interest force under a Markov chain interest rate. Yang & Zhang [12] considered a discrete-time insurance risk model by using an autoregressive process to model both the premuims and the claims, and they also included investment incomes in their model. Cai [1,2] investigated the ruin probabilities in two risk models, with independent premiums and claims and used a first-order autoregressive process to model the rates of interest. Cai & Dickson [3] obtained Lundberg inequalities for ruin probabilities in a two discrete-time risk process with a Markov chain interest model and independent premiums and claims. The author established Lundberg inequalities using a recursive technique for ruin probabilities in a two discrete-time risk process with homogenous Markov chain

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premiums when claims and rate of interest sequences are independent [5], and also by the Martingale approach in a two discrete-time risk process with homogenous Markov chain claims when premiums and rate of interest sequences are independent [6].

In this article, we extend the models considered by Cai & Dickson [3] to introduce homogenous Markov chain claims and homogenous Markov chain premiums, assuming independent rates of interest.

2. The Model and Basic Assumptions

Let $X = \{X_n\}_{n\geq 0}$ denote the premiums, $Y = \{Y_n\}_{n\geq 0}$ the claims and $I = \{I_n\}_{n\geq 0}$ the interests, where X, Y and I are defined on the probability space (Ω, A, P). To establish the probability inequalities for ruin probabilities, two styles of premium collection are considered. On the one hand, for premiums collected at the beginning of each period, the surplus process $\{U_n^{(1)}\}_{n\geq 1}$ with initial surplus u can be written

$$U_n^{(1)} = U_{n-1}^{(1)}(1+I_n) + X_n - Y_n\,, \tag{2.1}$$

which can be rearranged as

$$U_n^{(1)} = u\prod_{k=1}^{n}(1+I_k) + \sum_{k=1}^{n}\left(X_k - Y_k\right)\prod_{j=k+1}^{n}(1+I_j)\,. \tag{2.2}$$

On the other hand, for premiums collected at the end of each period, the surplus process $\{U_n^{(2)}\}_{n\geq 1}$ with initial surplus u is

$$U_n^{(2)} = (U_{n-1}^{(2)} + X_n)(1+I_n) - Y_n\,, \tag{2.3}$$

or equivalently

$$U_n^{(2)} = u\prod_{k=1}^{n}(1+I_k) + \sum_{k=1}^{n}\left[X_k(1+I_k) - Y_k\right]\prod_{j=k+1}^{n}(1+I_j)\,, \tag{2.4}$$

where throughout this article $\prod_{t=a}^{b} z_t = 1$ and $\sum_{t=a}^{b} z_t = 0$ if $a > b$.

We make several assumptions.

Assumption 2.1. $U_o^{(1)} = U_o^{(2)} = u > 0\,.$

Assumption 2.2. $X = \{X_n\}_{n\geq 0}$ is an homogeneous Markov chain, such that for any n the values of X_n are taken from a set of non-negative numbers $E_X = \{x_1, x_2, \cdots, x_m, \cdots\}$ with $X_0 = x_i$ and

$$p_{ij} = P\left[\,\omega\in\Omega : X_{m+1}(\omega) = x_j \middle| X_m(\omega) = x_i\right], \quad (m\in N), \quad x_i, x_j \in E_X\,,$$

where $0 \leq p_{ij} \leq 1\,,\ \sum_{j=1}^{+\infty} p_{ij} = 1\,.$