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Accelerated GPMHSS Method for Solving Complex Systems of Linear Equations

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Abstract. Preconditioned modified Hermitian and skew-Hermitian splitting method (PMHSS) is an unconditionally convergent iteration method for solving large sparse complex symmetric systems of linear equations, and uses one parameter α. Adding another parameter β, the generalized PMHSS method (GPMHSS) is essentially a two-parameter iteration method. In order to accelerate the GPMHSS method, using an unexpected way, we propose an accelerated GPMHSS method (AGPMHSS) for large complex symmetric linear systems. Numerical experiments show the numerical behavior of our new method.

AMS subject classifications: 65F10, 65W50

Key words: Preconditioning, complex systems of linear equations, PMHSS method, GPMHSS method, AGPMHSS method.

1. Introduction

Many applications in scientific computing and engineering can be transformed into solving the following large sparse and complex symmetric linear equations

$$Ax = b, \qquad A \in \mathbb{C}^{n\times n}, \qquad x, b \in \mathbb{C}^n, \tag{1.1}$$

where $A = W + iT$, W, $T \in \mathbb{R}^{n\times n}$ are symmetric matrices, with W positive definite and T positive semi-definite. Here and in the sequel, i denotes the imaginary unit. Such applications arise in quantum mechanics [23], diffuse optimal tomography [1], structural dynamics [15], FFT-based solution of certain time-dependent PDEs [12], molecular scattering [21], and lattice quantum chromodynamics [16], etc.

Generally, direct methods and iteration methods are two main classes of methods for solving systems of linear equations. Direct solution methods, such as Gaussian elimination,

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LU-decomposition, are often preferred to iterative methods because of their robustness and predictable behavior. However, when coefficient matrix A is very large and sparse, direct solvers may cost too much time and storage. Iterative methods, such as Krylov subspace methods, are easier to keep and exploit the sparsity of A, thereby require much less computer storage than direct methods, and implement efficiently on high-performance computers than direct methods. Thus, iteration methods have been widely concerned by scholars all the time, see [17, 19, 22] and references therein.

Based on the Hermitian and skew-Hermitian splittings, Bai, Golub and Ng [8] have proposed the Hermitian and skew-Hermitian splitting (HSS) method for non-Hermitian positive-definite linear systems. They have also proved that this method converges unconditionally to the exact solution of the system, and if it is used to solve the system of linear equations with Hermitian positive-definite coefficient matrix, the convergence speed is same as that of the conjugate gradient method. Owing to the effectiveness and robustness of the HSS method, it has received attentions from many scholars, eg. see [5–7, 10, 11]. Even some scholars used HSS-type methods as the inner iterative solver, and Newton-type methods as the outer iterative solver, proposed several effective methods for solving nonlinear equations, eg. [11, 13, 18, 20, 24, 26, 27].

Nevertheless, when A is complex, the convergence rate of each method referred above, reduces significantly since the resolution of the linear system (1.1) needs a complex algorithm. In order to overcome this deficiency, Bai *et al.* [2–4] proposed the modified HSS (MHSS) iteration and preconditioned modified HSS (PMHSS) to solve complex symmetric linear systems. Based on the PMHSS method, Xu [25] proposed its generalization for complex symmetric indefinite linear systems, while Mehdi *et al.* [14] presented the generalized preconditioned MHSS method (GPMHSS) for complex symmetric linear systems with two parameters. When the parameters satisfy some ordinary conditions, the GPMHSS iteration method can converge unconditionally with any initial vector.

In this paper, based on the GPMHSS method, we establish its successive-overrelaxation scheme. This work is organized as follows. In Section 2, we introduce the GPMHSS method due to Mehdi, Marzieh and Masoud [14]. In Section 3, we first give the corresponding fixed point equations of the GPMHSS method, and illustrate the equivalence between the new equations and (1.1). Then we propose an accelerated GPMHSS method (AGPMHSS) for (1.1). The theoretical analysis is given in Section 4. Numerical experiments are made in Section 5, which illustrate the numerical behavior of our new method.

2. The GPMHSS Method

In this section, we introduce the GPMHSS method [14] for solving large sparse and complex symmetric linear system (1.1). The splitting iteration method can be described as follows.

The GPMHSS iteration method [14]
Let $x_0 \in \mathbb{C}^n$ be an arbitrary initial guess. Compute x_{k+1} for $k = 0, 1, \cdots$ using the following