East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.130612.210213a

Vol. **3**, No. 1, pp. 27-47
February 2013

A Transformed Coordinates Shallow Water Model for the Head of the Bay of Bengal Using Boundary-Fitted Curvilinear Grids

Farzana Hussain*

Department of Mathematics, Shahjalal University of Science & Technology, Sylhet, Bangladesh.

Received 13 June 2012; Accepted (in revised version) 21 February 2013

Available online 28 February 2013

Abstract. The Bay of Bengal is surrounded by coastline except to the south, where there is open sea. The coastline bends most sharply along the coast of Bangladesh, and there are many small and large islands in the offshore region there. In order to incorporate the island boundaries and the curved coastline properly, in any numerical scheme it is often necessary to consider a very fine grid resolution along the coastal belts whereas this is unnecessary away from the coasts. However, a very fine resolution involves more memory and more CPU time in the numerical solution process, and invites numerical instability. On the other hand, boundary-fitted curvilinear grids in hydrodynamic models for coastal seas, bays and estuaries not only fit to the coastline but also render the finite difference schemes simpler and more accurate. In this article, the boundary-fitted curvilinear grids for the model represent the complete boundary of the area considered by four curves defined by four functions, and the four boundaries of two of the larger islands are then represented approximately by two general functions. An appropriate independent coordinate transformation maps the curvilinear physical area to a square domain, and each island boundary is transformed to a rectangle within this square domain. The vertically integrated shallow water equations are transformed to the new space domain, and solved by a regular explicit finite difference scheme. The model is applied to compute the water levels due to astronomical tides, and also the water levels due to surges associated with tropical storms that hit the coast of Bangladesh.

AMS subject classifications: 35F30, 37M05, 65N06, 65N22, 65N50

Key words: Tropical storms, surge, tide generation, shallow water equations, boundary-fitted grids, Bay of Bengal.

1. Introduction

Tropical storms pose the most destructive natural disasters for the coastal region of Bangladesh, and the associated surges are usually more dangerous than the storms themselves. Sometimes a surge may rise from 9 to 12 meters as it rushes towards the land [12],

*Corresponding author. *Email address:* `farzanahn@live.com` (F. Hussain)

where it causes very severe damage to life and property. There are various factors responsible for the large surge levels along the coast of Bangladesh — such as shallowness of water, off-shore islands, the bending of the coastline, oceanic bathymetry, low lying islands, huge discharge through the rivers, etc.. Moreover, the tidal range (the difference between the high and low tide) is large at the head Bay of Bengal, and if a storm approaches the coast at the time of a high tide the devastation is much worse.

Many analyses have been done for the coast of Bangladesh since 1972, after the major devastation from the November 1970 storm [1–13]. They are based on vertically integrated two-dimensional models and fall into two main categories — viz. stair step models and partially boundary-fitted transformed coordinate models. The coastal and island boundaries are approximated along the nearest finite difference gridlines of the numerical scheme in a stair step model, so its accuracy depends on the grid size. If the grid size is not small, the representation of the coastal boundaries is not accurate. In a boundary-fitted transformed coordinate model, the curvilinear boundaries are transformed into straight lines such that regular finite difference methods can be used in the transformed domain. The pioneering works of Das [1] and Flierl & Robinson [2] were linear stair step models where the friction coefficient was neglected. The stair step model of Das *et al.* [3] was an extension of Das [1], to simulate tidal and surge interactions along the east coast of India and the coast of Bangladesh. The nonlinear stair step model of Johns & Ali [4] included the major rivers and islands, in simulating tidal and surge interaction along the coast of Bangladesh. This model was also used to verify the effect of cyclone tracks and islands on surge levels, and to compute the extent of inland flooding. However, a very fine resolution could not be considered, so the representation of the coastal boundaries in these models was not accurate. To more acurately account for bending coastlines and offshore islands, Roy [12] developed a fine mesh numerical scheme for the Meghna estuary nested into a coarse mesh scheme extending up to 15°N latitude. In the fine mesh component, all of the major islands were incorporated through a proper stair step representation. This model is similar to that of Johns *et al.* [9] for the East Coast of India and the coast of Bangladesh. Johns *et al.* [7] first used partially boundary-fitted curvilinear grids in a transformed coordinate model for the east coast of India to simulate the surge generated by the Andra Cyclone of 1977. In their scheme, the east coast of India and the western open sea boundary were considered as curves represented by two functions, and the north and south open sea boundaries were represented by straight lines. The spatial coordinate transformation they applied mapped the physical domain into a rectangular one. Dube *et al.* [5] represented the natural shoreline of the Bangladesh coast by a curvilinear boundary and used boundary-fitted curvilinear grids. A transformation similar to that of Johns *et al.* [7] was applied in order to make the computational domain rectangular, where an explicit finite difference scheme was used to integrate the shallow water equations. Roy [11] used the model of Dube *et al.* [5] to test the sensitivity of the surge level due to various meteorological and oceanographic factors along the Bangladesh coast. Dube *et al.* [6], Johns *et al.* [8, 9] and Sinha *et al.* [10] represented coastlines by curvilinear boundaries and also invoked coordinate transformations, but did not incorporate any offshore islands. The main difficulty in incorporating the islands was that the whereabouts of their boundaries