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Partial Eigenvalue Assignment for Undamped Gyroscopic Systems in Control

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Abstract. The solvability of a partial eigenvalue assignment problem for undamped gyroscopic control systems is proved and its explicit solutions are found. In addition, the problem of replacing certain eigenvalues by required ones while keeping the remaining eigenpairs unchanged, is solved by a multi-step method. The method is easily implementable and does not involve receptance matrices and Sylvester equation solutions. Numerical examples show the efficiency of the method.

AMS subject classifications: 65F18, 93C15

Key words: Inverse problem, gyroscopic system, partial eigenvalue assignment, multi-step method.

1. Introduction

A gyroscope is a rotating body with one symmetry axis such that the rotation about the symmetry axis is comparatively faster than the rotation about any other axis. In modern usage, a gyroscope is a system consisting of a symmetric rotor, which spins rapidly about the symmetry axis and free to move about one or two perpendicular axes. A considerable number of spinning bodies can be regarded as gyroscopes — e.g. helicopter rotor blades or spin stabilised satellites with elastic appendages such as solar panels or antennas. It is therefore no surprise that the gyroscopic systems attracted substantial attention [9]. In particular, the vibrating phenomenon of the undamped gyroscopic systems, such as the rotors of generators, solar panels on the satellite and so on may be modeled by the following second-order ordinary differential system

$$M\ddot{z}(t)+G\dot{z}(t)+Kz(t)=0, \tag{1.1}$$

where $z(t)\in\mathbf{R^n}$ and $M,G,K\in\mathbf{R^{n\times n}}$ are, respectively, mass, gyroscopic and stiffness matrices [12]. Note that the matrix M is assumed to be symmetric positive definite, G skew-symmetric, K symmetric nonsingular and the time derivatives of z are, respectively, the

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vectors of displacement, velocity and acceleration. The associated open-loop pencil is given by $P(\lambda) = \lambda^2 M + \lambda G + K$.

To overcome undesirable effects of vibrations caused by certain eigenvalues of the system, one can use a suitable control force to reassign those eigenvalues without changing the others. In control theory, this problem is known as the partial pole placement problem [5]. We consider the control force of the form

$$f = Bu(t),$$

where $B \in \mathbf{R}^{\mathbf{n}\times\mathbf{m}}$ is a full column rank control matrix and $u(t) \in \mathbf{R}^{\mathbf{m}}$ is a control vector.

In order to assign the eigenvalues, an active control based on the velocity and displacement state feedback can be employed. To improve the performance of second-order linear systems, Carvalho [3] used estimators, sensors and actuators feedback, and for partial eigenstructure assignment of undamped vibration systems, Zhang [22] adopted acceleration and displacement feedback. The later approach is even more important and attractive because of frequent appearance of the accelerometers in practice and since acceleration and velocity are easier to measure [17]. Thus we consider the control vector $u(t)$ of the form

$$u(t) = F_1^\top \ddot{z}(t) + F_2^\top \dot{z}(t) + F_3^\top z(t),$$

where $F_1, F_2, F_3 \in \mathbf{R}^{\mathbf{n}\times\mathbf{m}}$ are acceleration, velocity and displacement state feedback matrices, respectively. If $F_1 = 0$, $u(t)$ is the common control vector using velocity and displacement state feedback, then the closed-loop system corresponding to (1.1) is

$$\left(M - BF_1^\top\right)\ddot{z}(t) + \left(G - BF_2^\top\right)\dot{z}(t) + \left(K - BF_3^\top\right)z(t) = 0. \tag{1.2}$$

Mathematically, the partial eigenvalue assignment problem for undamped gyroscopic control systems is to find matrices $F_1, F_2, F_3 \in \mathbf{R}^{\mathbf{n}\times\mathbf{m}}$ such that a few eigenvalues of the closed-loop pencil

$$P_c(\lambda) = \lambda^2\left(M - BF_1^\top\right) + \lambda\left(G - BF_2^\top\right) + \left(K - BF_3^\top\right)$$

are altered as required and the resting eigenpairs remain unchanged — i.e. possessing the no spill-over property [20–24]. This leads to the partial eigenvalue assignment problem of undamped gyroscopic control systems (GPEAP).

GPEAP Problem. Let $M, G \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$ and $K \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$ be, respectively, symmetric positive definite, skew-symmetric and symmetric nonsingular matrices and let $B \in \mathbf{R}^{\mathbf{n}\times\mathbf{m}}$ be a full column rank control matrix. For a self-conjugate subset $\{\lambda_k\}_{k=1}^p$, $p < 2n$ of open-loop eigenvalues $\{\lambda_k\}_{k=1}^{2n}$, the corresponding set of eigenvectors $\{x_k\}_{k=1}^p$ and a self-conjugate set $\{\mu_k\}_{k=1}^p$, the GPEAP Problem consists in finding state feedback matrices $F_1, F_2, F_3 \in \mathbf{R}^{\mathbf{n}\times\mathbf{m}}$ such that the closed-loop pencil

$$P_c(\lambda) = \lambda^2\left(M - BF_1^\top\right) + \lambda\left(G - BF_2^\top\right) + \left(K - BF_3^\top\right)$$

has the eigenvalues $\{\mu_k\}_{k=1}^p$ and eigenpairs $\{\lambda_k, x_k\}_{k=p+1}^{2n}$.