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Quadrilateral Cell-Based Anisotropic Adaptive Solution for the Euler Equations

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Abstract. An anisotropic solution adaptive method based on unstructured quadrilateral meshes for inviscid compressible flows is proposed. The data structure, the directional refinement and coarsening, including the method for initializing the refined new cells, for the anisotropic adaptive method are described. It provides efficient high resolution of flow features, which are aligned with the original quadrilateral mesh structures. Five different cases are provided to show that it could be used to resolve the anisotropic flow features and be applied to model the complex geometry as well as to keep a relative high order of accuracy on an efficient anisotropic mesh.

AMS subject classifications: 51E12, 65M08, 68U05, 68U20

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1 Introduction

Fluid phenomena contain many different length scales and flow features, such as flow separation, shock waves, interfaces between different fluids and viscous boundary layers. To resolve all these scales and features, the mesh adaption is a natural and economical way for accurate and efficient solutions. As a result, a large number of adaptive algorithms have been proposed in the literature.

Among them, the Cartesian adaptive method [1–14] is a quite popular method in which the nested hierarchical data structure is often employed. The main feature of this kind of methods is that the global index of a cell could be calculated by the level and the

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local index. This reduces the memory requirement. Most of them are isotropic Cartesian method which uses the tree data structures such as the quad-tree and the oct-tree. These isotropic Cartesian grid methods could not be efficient for the flow problems where highly anisotropic meshes aligned with the flow direction are most appropriate. Hence, several anisotropic Cartesian adaptive grid methods [11–14] are developed. Wang et al. [11, 12] proposed an anisotropic grid generation by adopting a 2^N tree data structure, which is a natural extension of the quad-tree or oct-tree. Ham et al. [13] proposed another anisotropic adaptive method. Like the isotropic Cartesian grid methods, the global index of a cell for this anisotropic Cartesian method [13] could be obtained by the level in each direction and the local index.

One of the main drawbacks of all these adaptive Cartesian grid methods is their difficulty when they are applied to complex geometry with non-Cartesian boundaries. To tackle this problem, some researchers [9] adopted the cut-cell or hybrid grid technique. These will increase the complexity and reduce the main advantage of the Cartesian method.

Another type of anisotropic adaptive method to treat the complex geometry is the unstructured mesh based adaptive methods [15–18]. Most of them are the triangular or tetrahedral cell based adaptive method. The adaptive grid based on triangular mesh (or tetra-hedras in 3D) is often results in highly skewed cells for anisotropic flow features such as boundary layers, which makes it difficult to maintain high accuracy. Besides, successive refinements in one direction of the triangle based adaptive method create triangle cell with some very small angles between faces. This resulted in large mesh skewness which can cause severe degradation in both solution accuracy and convergence.

In order to address the issues of mesh efficiency and solution convergence for adaptive solutions, Qin and Liu [19] proposed a feature-aligned mesh adaptation method, which combined the flexibility of unstructured meshes for complicated geometries and structured feature aligned mesh blocks to achieve efficient and accurate mesh adaptation. The refinement only needs to be carried out normal to the flow features, leaving relatively large mesh spacing along the features, such as shock waves and boundary layers. The overall solution efficiency is substantially improved by (1) the flow feature aligned anisotropic adaptive mesh and (2) the better behavior of the solver's convergence on such meshes. However, the extraction of the flow features and the insertion of the structured blocks for complicated flow problems are key issues to be fully addressed and automated.

As a step towards a fully automated and robust feature aligned mesh adaptation, this paper proposes an unstructured quadrilateral-cell based anisotropic mesh adaptive method to resolve the anisotropic flow features, given a background initial body conforming mesh. As compared to the Cartesian anisotropic adaptive method [11–14], the fully unstructured quadrilateral-cell based anisotropic adaptive meshes are body fitted rather than the cut-cell method or the hybrid method [9]. Similar to the isotropic unstructured quadrilateral based adaption proposed by Sun and Takayama [17] and Zheng et al. [18], the neighboring cells of each edge and the sub-edges of each edge are stored. However, the neighboring cells for the mother edge may not include the cell that the edge