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Balanced Monitoring of Flow Phenomena in Moving Mesh Methods

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Abstract. Adaptive moving mesh research usually focuses either on analytical derivations for prescribed solutions or on pragmatic solvers with challenging physical applications. In the latter case, the monitor functions that steer mesh adaptation are often defined in an ad-hoc way. In this paper we generalize our previously used monitor function to a balanced sum of any number of monitor components. This avoids the trial-and-error parameter fine-tuning that is often used in monitor functions. The key reason for the new balancing method is that the ratio between the maximum and average value of a monitor component should ideally be equal for all components. Vorticity as a monitor component is a good motivating example for this. Entropy also turns out to be a very informative monitor component. We incorporate the monitor function in an adaptive moving mesh higher-order finite volume solver with HLLC fluxes, which is suitable for nonlinear hyperbolic systems of conservation laws. When applied to compressible gas flow it produces very sharp results for shocks and other discontinuities. Moreover, it captures small instabilities (Richtmyer-Meshkov, Kelvin-Helmholtz). Thus showing the rich nature of the example problems and the effectiveness of the new monitor balancing.

AMS subject classifications: 35L65, 65M50, 74S10, 76L05, 76N15

Key words: Moving mesh method, conservative interpolation, balanced monitor function, directional adaptation, hydrodynamics, implosion problem.

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1 Introduction

Adaptive mesh methods improve local resolution of numerical solvers and, as a result, improve their performance. Results are significantly sharper than those obtained by using a uniform mesh with more mesh points. True gain in performance is only obtained,

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though, when the adaptive methods perform well automatically. This requires a balanced monitoring of flow phenomena, without manual fine-tuning of parameters by trial and error. This paper presents such a balanced monitoring, combined with a powerful finite volume solver, applied to hydrodynamical problems.

Adaptivity Three types of adaptive methods are generally distinguished: *h*-, *r*- and *p*-refinement. The *h*-refinement or *local refinement* splits mesh cells into smaller ones based on some criterion. This can provide great levels of detail and is widely used in CFD-codes. The implementation is nontrivial due to the hierarchical structure of the domain discretisation. The eventual number of mesh cells is sometimes hard to predict, which may lead to unexpectedly long running times. Although the initial structure of the mesh fixes the shape and orientation of the mesh cells, e.g., rectangular, the unlimited amount of possible refinement makes these methods very powerful. In one of our experiments (Section 5) we will make a comparison between our *r*-refinement results and the *h*-refinement results produced by AMRVAC [23,36].

The *r*-refinement or *(adaptive) moving mesh refinement* moves mesh points towards regions that need refinement based on some criterion. The number of points remains constant, which gives fairly predictable running times. Besides, the mesh cells can change shape, position and orientation, so that alignment with, e.g., shocks or vortices is well possible. For specific problems, the fixed number of mesh points may impose a limit on the achievable resolution. This paper deals with *r*-refinement only and shows that it can achieve great levels of detail. Tang [32] provides an extensive historical overview of moving mesh methods and their applications in CFD. Zegeling [43] presents Winslow-type adaptivity applied to a wide range of problems. We also give a detailed and systematic overview [35] on Winslow-type adaptivity, harmonic maps, geometric conservation laws and more related methods.

The combination of the above two methods, called *hr*-refinement combines the advantages of both methods and is occasionally used, e.g., by Lang et al. [20] and Anderson et al. [1].

The *p*- (and *hp*-) refinement is generally applied in different frameworks than what we consider here. It involves the local increase of polynomial order of the basis functions in finite element methods.

Moving mesh research We employ a variational formulation of mesh adaptation, an approach which has become well-known over the past five decades. A short historic overview of moving mesh methods is given in Section 3.3.1. Tang and Tang [30] presented a moving mesh algorithm in a pragmatic combination with a finite volume solver. Over the past five years, this inspired several others. The technique is usually applied to hydrodynamics (HD), e.g., by Tang [31] and Zegeling et al [44], and to magnetohydrodynamics (MHD), e.g., by Han and Tang [13], Tan [27], Van Dam and Zegeling [39] and Zegeling [42]. Moving mesh methods generally have little dependency on the physical PDEs under consideration, as diverse applications show, e.g., the Navier-Stokes equations by Di et al. [11] and the Hamilton-Jacobi equations by Tang et al. [29]. A similar