COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **6**, No. 3, pp. 655-672

Commun. Comput. Phys.
September 2009

Hydrophobic Effect in a Continuum Model of the Lipid Bilayer

Phillip L. Wilson[1,*], Huaxiong Huang[2] and Shu Takagi[3]

[1] *Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand.*
[2] *Department of Mathematics and Statistics, York University, Toronto, M3J 1P3, Canada.*
[3] *Department of Mechanical Engineering, The University of Tokyo, Tokyo, Japan.*

Received 16 August 2008; Accepted (in revised version) 24 December 2008

Available online 19 February 2009

Abstract. We study a continuum paradigm of the lipid bilayer based on minimizing the free energy of a mixture of water and lipid molecules. This paper extends previous work of Blom and Peletier [European J. Appl. Math., 15 (2004), pp. 487-508] in the following ways. (a) It formulates a more general model of the hydrophobic effect to facilitate connections with microscale simulations and first-principles analysis. (b) It clarifies the meaning and role of the model parameters. (c) It outlines a method for determining parameter values so that physically-realistic bilayer density profiles can be obtained, for example for use in macroscale simulations. Points (a)-(c) suggest that the model has potential to robustly connect some micro- and macroscale levels of multiscale blood flow simulations. The mathematical modelling in point (a) is based upon a consideration of the underlying physics of inter-molecular forces. The governing equations thus obtained are minimized by gradient flows via a novel numerical approach; this enables point (b). The numerical results are shown to behave physically in terms of the effect of background concentration, in contrast to the earlier model which is shown here to not display the expected behaviour. A "short-tail" approximation of the lipid molecules also gives an analytical tool which yields critical values of some parameters under certain conditions. Point (c) involves the first quantitative comparison of the numerical data with physical experimental results.

AMS subject classifications: 74G15, 74G65, 74K15, 74L15

Key words: Lipid bilayer, free energy, continuum model, numerical minimization.

*Corresponding author. *Email addresses:* p.wilson@math.canterbury.ac.nz (P. L. Wilson), hhuang@mathstat.yorku.ca (H. Huang), takagi@mech.t.u-tokyo.ac.jp (S. Takagi)

1 Introduction

We show that a continuum paradigm [1] of the lipid bilayer can produce physically-realistic bilayer properties. We first introduce into the paradigm a new model of the hydrophobic effect. Secondly, we investigate the influence and meaning of the paradigm's parameters by performing the first quantitative comparison of numerical solutions of the paradigm with physical experimental data, and in so doing provide a method for determining parameter values.

As is well-known, the cell is the fundamental element of all living matter. The activity of the cell sustains life and the cell itself is sustained by a metabolism which utilizes mass transfer through its membrane. The cell membrane is composed of a double layer of lipid molecules (lipids) with proteins and other components floating in it [9]. The dynamics of this lipid bilayer membrane become especially important in the case of dispersed components in the blood, such as red blood cells (RBCs), white blood cells, platelets and so on, because the deformation dynamics of these membranes directly affect the mass transfer in the blood. These membranes are often modelled as hyperelastic due to the presence of a cytoskeleton. On the other hand, a liposome, composed of lipid bilayers only, is usually modelled as a two-dimensional fluid membrane, because the membrane lipid molecules can easily move laterally within the bilayer. Liposomes are used as drug delivery agents (DDAs) and artificial oxygen-carriers in blood. Although fluid, a "soft" entropic force called the hydrophobic force (or effect) gives the pure lipid bilayer integrity [3].

In all these cases the behavior of the lipid bilayer is responsible for the mass transfer through the cell membranes. Hence, the modelling of the lipid bilayer membrane from a molecular level through to the continuum level is expected ultimately to predict mass transfer behavior in blood [2, 8, 10, 11].

Here we focus on one crucial intermediate scale the mesoscale at which both molecular physics and continuum mechanics are important to the bilayer dynamics, since the bilayer is typically only two molecules in thickness but extends laterally for several micrometers. Understanding how membrane composition affects deformability, and how deformation affects the mass-transfer properties of RBCs and DDAs, are key to the multiscale modelling of blood flow. In this paper, we consider bilayers composed of one type of lipid.

[1] base their continuum paradigm, herein called the "BP paradigm", on the mesoscopic dynamics framework of [5], minimizing a free energy for a system of lipid and water molecules. Formally, the intrinsic free energy of the system is minimized with respect to a constraint that the (unobservable) distribution of the molecules generates the (observable) continuous volume fractions, thus assuming that the microstate has relaxed to equilibrium over the relatively long time scale of the continuous description.

Within this paradigm, several different models of lipid structure and inter-molecular interaction may be considered, and here we present a new interaction model which differs from the original choice in [1] in a number of ways. Our main point is a new model of the hydrophobic effect which has two advantages over the original, both stemming from