

## Numerical Study on Two-Dimensional Micro-Channel Flows Using the Gas-Kinetic Unified Algorithm

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Received 2 December 2016; Accepted (in revised version) 7 June 2017

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**Abstract.** Based on the Boltzmann model equation, the Gas-Kinetic Unified Algorithm (GKUA) will be developed to simulate the two-dimensional micro-scale gas flows with irregular configuration. The numerical scheme for the direct evaluation of the unified velocity distribution function in the computable model of the Boltzmann equation and the multi-block grid docking technology are constructed, and the numerical procedures of characteristic-based boundary conditions are presented to model the gas-surface interaction and the inlet/outlet boundaries for the two-dimensional micro-channel flows. The two-dimensional Couette flow, the pressure-driven micro-channel flows, and the irregular micro-orifice flows in different scales are numerically solved from high rarefied free-molecule to near-continuum flow with the Knudsen numbers of  $Kn = 100 - 0.01$ . The computed results are compared and validated with the DSMC data in the transitional flow regime and the slip N-S solutions in the near-continuum flow regime, in which the GKUA is verified accurately and smoothly to simulate the two-dimensional micro-channel flows with strong adaptability and good precision. The micro-channel flow features with the wide range of  $Kn$  numbers in the near-continuum slip and transitional flow regimes are revealed, and it is probable to provide a way in developing a new numerical algorithm for micro-scale flows.

**AMS subject classifications:** 82B40, 82C40, 82D05

**Key words:** Micro-channel flow, Boltzmann model equation, gas molecular velocity distribution function, gas-kinetic unified algorithm, non-equilibrium rarefied effect of micro-scale flow.

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## 1 Introduction

With the development of MicroElectroMechanical Systems (MEMS), the researches on micro-scale flows are gradually deepened and applied to the micro flow system in many disciplines such as biology, medication, optics, aerospace mechanics and electronic engineering et al. [1–6]. The gas flows from MEMS in configuration of microns or less may be characterized by relatively large Knudsen numbers,  $Kn = \lambda/L$ , which is defined as the ratio of the molecular mean free path  $\lambda$  to the characteristic length  $L$ . When the molecular mean free path remains unchanged, if the characteristic length is small enough, the flow has higher  $Kn$  number, and the  $Kn$  of micro-scale flow can reach the order of magnitude of 1. The flows in MEMS are most likely in the slip flow  $10^{-3} \leq Kn \leq 10^{-1}$  and transitional flow  $10^{-1} \leq Kn$  regimes [3,7–10]. In the micro-scale world, the non-continuum nonlinear viscosity and rarefied non-equilibrium effect become prominent, which may make the MEMS behave quite differently from their counterparts in macroscopic scale [3,4,10].

Because of the high rarefied flow effect of micro scale, the numerical method of the traditional Navier-Stokes equation appears large error, and the N-S solver with the slip boundary modification can be developed [3, 11, 12], but when the  $Kn$  number is up to about 0.1, even the wall-slip N-S equation is also very difficult or invalid for the micro flows. On the other hand, the Direct Simulation Monte Carlo (DSMC) method [13–18] can always get good results of flow characteristics in the rarefied regime, but due to the high number density and low flow velocity of the MEMS, the results of DSMC appear so larger statistical fluctuations that the real useful information is sometimes even be submerged. So traditional methods are very difficult to get an accurate result for the low velocity and micro scale flow with high  $Kn$  numbers.

The Boltzmann equation can describe the molecular transport phenomena for the full spectrum of flow regimes and act as the main foundation for the study of complex gas dynamics. By using the kinetic model equation to simplify the collision integral term of the Boltzmann equation, a new class of methods based on the mesoscopic theory connecting macroscopic fluid dynamics and microscopic molecular dynamics has been proposed and developed for low-speed micro-scale gas flows. Notable examples include the Discrete Velocity Models (DVM) [19,20], the Lattice Boltzmann method (LBM) [21–26] and the Gas-Kinetic Scheme (GKS) [27–29]. These methods are closely related to the kinetic theory or the Boltzmann-BGK-type equation and thereby substantially extend the analytical and computational capabilities to low-speed gas flows. Specially, it can be indicated in [9,30–39] that a unified computational modeling based on the Boltzmann equation has been presented in describing flow transport phenomena around complex bodies in various flow regimes, and the theory and computational techniques of a gas-kinetic unified algorithm [9,32–35,37,38,40,41] have been established and used to simulate the aerothermodynamics from highly rarefied free-molecule flow to continuum flow regimes.

This study is aimed at extending and applying this gas kinetic method to simulate the micro-scale gas flows. Considering that two-dimensional or axisymmetrical micro-channels are typical components of conventional MEMS, there are many examples of