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An Efficient Immersed Boundary-Lattice Boltzmann Method for the Simulation of Thermal Flow Problems

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Abstract. In this paper, a diffuse-interface immersed boundary method (IBM) is proposed to treat three different thermal boundary conditions (Dirichlet, Neumann, Robin) in thermal flow problems. The novel IBM is implemented combining with the lattice Boltzmann method (LBM). The present algorithm enforces the three types of thermal boundary conditions at the boundary points. Concretely speaking, the IBM for the Dirichlet boundary condition is implemented using an iterative method, and its main feature is to accurately satisfy the given temperature on the boundary. The Neumann and Robin boundary conditions are implemented in IBM by distributing the jump of the heat flux on the boundary to surrounding Eulerian points, and the jump is obtained by applying the jump interface conditions in the normal and tangential directions. A simple analysis of the computational accuracy of IBM is developed. The analysis indicates that the Taylor-Green vortices problem which was used in many previous studies is not an appropriate accuracy test example. The capacity of the present thermal immersed boundary method is validated using four numerical experiments: (1) Natural convection in a cavity with a circular cylinder in the center; (2) Flows over a heated cylinder; (3) Natural convection in a concentric horizontal cylindrical annulus; (4) Sedimentation of a single isothermal cold particle in a vertical channel. The numerical results show good agreements with the data in the previous literatures.

AMS subject classifications: 76M28, 76R05, 76R10

Key words: Immersed boundary method, lattice Boltzmann method, thermal flow, Dirichlet boundary condition, Neumann boundary condition, Robin boundary condition, Taylor Green vortices.

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1 Introduction

The treatment of boundary conditions directly affects the accuracy and stability of the numerical simulation. How to resolve complex boundaries accurately and efficiently is a great challenging issue for CFD researchers. There are a large number of flow and heat transfer problems in irregular regions in the fields of science and engineering. Currently there exist two kinds of categories to implement the boundary conditions in the complex domains: body-fitted and fixed grid methods. The key issues of body-fitted methods are mapping the irregular regions to the regular domains by using algebraic and differential methods and then solving the governing equations in the structure meshes. These methods which couple boundary conditions and governing equations are extremely difficult to solve. When resolving moving boundaries in fluid-structure interactions, body-fitted methods require the complex grid remapping at every time step which may involve very large computational costs and cause loss of the computational accuracy. Fixed grid methods are another kinds of techniques to treat the complex boundary problems. The basic idea is to extend the computational domain to a simpler computational domain, and the boundary conditions can be enforced on the boundary. In the CFD simulations, three different fixed grid techniques are very popular in recent years: distributed Lagrange multiplier/fictitious domain method (DLM/FDM), immersed interface method (IIM) and immersed boundary method (IBM).

The DLM/FDM was firstly introduced by Glowinski which is based on the finite element method [1, 2]. The main features of DLM/FDM are that the governing equations are discretized in space using a fixed grid, and the boundary conditions on the original domain can be enforced by applying the Lagrange multiplier technique. Glowinski *et al*. had successfully implemented this method to simulate the incompressible particulate flows. Later the DLM/FDM was extended to deal with heat transfer problems [3, 4]. The immersed interface method is also a fixed grid method by adopting a Cartesian grid. The IIM was proposed by Leveque and Li which incorporates the jump conditions into finite difference or finite volume schemes [5]. This method establishes the relationships between jumps and singular forces. And it computes the correction terms in the discretized governing equations to obtain the high accuracy solutions. The IIM has been used in the Stokes problem, Navier-Stokes flows, discontinuous viscosity problem [6–8].

In contrast to the fictitious domain method and immersed interface method, the immersed boundary method (IBM) which is also named the discrete delta function approach is used more widely for the interface problems. As early as 1970s, the original IBM was developed to study the blood flow problem in the heart by Peskin [9]. The method has been extended to many fluid and biological problems in complex geometries. The IBM replaces singular Dirac function by a smoother regularization function. The governing equations are discretized and solved on a fixed Eulerian grid and the interface is represented by a set of Lagrangian points. The forces at the Lagrangian points are spread to the near Eulerian points. When resolving the moving boundary problems, we only track the positions of the Lagrangian points. So the IBM reduces the computa-