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Efficient Operator Marching Method for Analyzing Crossed Arrays of Cylinders

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Abstract. Periodic structures involving crossed arrays of cylinders appear as special three-dimensional photonic crystals and cross-stacked gratings. Such a structure consists of a number of layers where each layer is periodic in one spatial direction and invariant in another direction. They are relatively simple to fabricate and have found valuable applications. For analyzing scattering properties of such structures, general computational electromagnetics methods can certainly be used, but special methods that take advantage of the geometric features are often much more efficient. In this paper, an efficient method based on operators mapping electromagnetic field components between two spatial directions is developed to analyze structures with crossed arrays of circular cylinders. The method is much simpler than an earlier method based on similar ideas, and it does not require evaluating slowly converging lattice sums.

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Key words: Periodic structure, Photonic crystal, Operator marching, Scattering problem.

1 Introduction

Periodic structures such as diffraction gratings [1–3], photonic crystals [4] and metamaterials [5], have many interesting and useful properties, and are very important in modern photonics technologies. In this paper, we consider three-dimensional (3D) biperiodic structures consisting of crossed arrays of cylinders. More precisely, we assume the structures are periodic in the x and y directions, and bounded by homogeneous media in the z direction, where $\{x,y,z\}$ is a Cartesian coordinate system. In the z direction, it is further assumed that the structure can be divided into a number of layers, where each layer

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is a periodic array of cylinders parallel to the x or y axis. Such crossed arrays of cylinders are relatively easy to fabricate, and appear as special 3D photonic crystals [6,7] and cross-stacked gratings [8].

For biperiodic structures bounded by homogeneous media in the z direction, a fundamental problem is to compute the transmitted and reflected waves for a given plane incident wave [9–12]. For electromagnetic waves and light, the governing equation is the Maxwell's equations. General numerical methods such as the finite element method [13–16], can always be used, but it is possible to develop more efficient special computational methods by taking advantage of the available geometric features. For crossed arrays of cylinders, each layer is invariant in one spatial variable. In an x-invariant layer, if the wave field is decomposed into Fourier modes in x, then these modes are uncoupled within the layer. This implies that although the problem is truly three dimensional, the wave field in each layer can be resolved by solving a sequence of two-dimensional (2D) problems corresponding to different Fourier modes in one spatial direction. If the structure has many layers in the z direction and some of these layers are identical, it is desirable to avoid duplicated calculations in identical layers. If the cylinders have circular cross sections, it is also possible to develop efficient semi-analytic method based on cylindrical wave expansions.

For crossed arrays of circular cylinders, a semi-analytic method based on scattering matrices and cylindrical wave expansions was developed by Smith *et al.* [17] and Yasumoto and Jia [18]. The scattering matrices are used to avoid duplicated computations in identical layers. Cylindrical wave expansions are used for calculating the scattering matrix of each layer. In both works, sophisticated lattice sums techniques [19] are needed to take care of the periodic array of cylinders in each layer. For the same structures, we developed a semi-analytic method based on Dirichlet-to-Neumann (DtN) maps [20]. The DtN-map approach also uses cylindrical wave expansions, but it avoids the tedious lattice sums and is more efficient.

In this paper, we develop a simpler and more efficient semi-analytic method for analyzing crossed arrays of circular cylinders. Instead of the DtN maps, we use operators that map two tangential components of the electromagnetic field to two different tangential components (the T2T operators). Since the tangential components are continuous across a surface, the T2T operators are also continuous across interfaces. This leads a number of simplifications compared with the DtN-map method [20]. Cylindrical wave expansions are also used, but as in the DtN-map method, lattice sums are not needed, since the T2T operators are constructed starting from the unit cells. Numerical examples are used to validate the method and illustrate its accuracy and efficiency.

2 Formulation

For linear electromagnetic waves in isotropic media without sources, the governing equations in the frequency domain are the Maxwell's equations: