Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2015-0004

Vol. **21**, No. 4, pp. 1118-1140
April 2017

Numerical Modeling of Material Points Evolution in a System with Gravity

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Received 30 October 2015; Accepted (in revised version) 30 March 2016

Abstract. The evolution of material points interacting via gravitational force in 3D space was investigated. At initial moment points with masses of 2.48 Sun masses are randomly distributed inside a cube with an edge of 5 light-years. The modeling was conducted at different initial distributions of velocities and different ratios between potential and kinetic energy of the points. As a result of modeling the time dependence of velocity distribution function of points was obtained. Dependence of particles fraction which had evaporated from initial cluster on time for different initial conditions is obtained. In particular, it was obtained that the fraction of evaporated particles varies between 0,45 and 0,63.

Mutual diffusion of two classes of particles at different initial conditions in the case when at initial moment of time both classes of particles occupy equal parts of cube was investigated.

The maximum Lyapunov exponent of the system with different initial conditions was calculated. The obtained value weakly depends on the ratio between initial kinetic and potential energies and amounts approximately 10^{-5}. Corresponding time of the particle trajectories divergence turned out to be 40-50 thousand years.

PACS: 05.20.-y, 05.70.Ln, 04.40.-b

Key words: Systems with gravitation, Lyapunov exponent, material points, velocity distribution function.

1 Introduction

Studying of dynamics of non-stationary gravitating systems is an actual problem for today. Such systems include, for example, stars, star clusters, galaxies and other massive objects.

Mutual influence of bodies interacting on the basis of gravitational field leads to the chaotization of their orbits. Between gravitating bodies correlations exist, which is caused

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by the random convergence of bodies as a consequence of which the significant change of their trajectories can occur. As a result of substantial convergence, bodies (material points) can gain enough energy to leave the system.

The behavior in time of a large number of bodies interacting through gravitational potential can be described by distribution functions. The problem is that the evolution of the distribution function for the system of bodies with gravity depends significantly on the initial conditions. At the moment, this relationship has not been studied enough.

Thus, the evolution of the distribution function of material points in three-dimensional space is an urgent task. The aims of this article are

- On the basis of numerical simulations to investigate the evolution of the system at different initial conditions (different initial ratios of kinetic and potential energies of the system, the initial distribution of the coordinates and velocities) and to obtain the time dependence of the particle distribution function and the time dependence of the fraction of evaporated particles from different initial conditions;
- to investigate the mechanisms of mixing of classes of particles in the system at different initial conditions;
- assess the degree of chaos as a result of the evolution of the system through the maximum Lyapunov exponent.

2 Dynamics of the systems with gravity

Systems with gravity relate to systems, which are based on long-range potential. The additivity of internal energy of the system is not typical for these systems, because the interaction with distant parts of the system cannot be neglected. As a result, such systems may stay in the long-lived quasi-stationary states, and the term "local equilibrium" for these systems cannot be introduced. Quasi-stationary states are reached in the process of collisionless relaxation.

One of the main postulates of thermodynamics states that the entropy and energy should be additive with respect to subsystems – that is, the interfacial contribution should be negligible. For systems with short-range forces this condition is fulfilled – in the thermodynamic limit interfacial energy is much less than the energy of the bulk (volume) of the substance. This, however, does not refer to systems with long-range forces, for which the interfacial volume cannot be clearly defined – every particle interacts with every other particle system, so that there is no clear separation of the volume and the border (see, e.g., [1]).

In the absence of correlations, the dynamic evolution of the distribution function of the system with long-range interactions is determined by collisionless equation of Boltzmann-Vlasov. One of its most important features is that the system described in this equation does not come into a state of thermodynamic equilibrium (in contrast with systems with short-range potentials).