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A High-Order Time Domain Discontinuous Galerkin Method with Orthogonal Tetrahedral Basis for Electromagnetic Simulations in 3-D Heterogeneous Conductive Media

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Abstract. We present a high-order discontinuous Galerkin (DG) method for the time domain Maxwell's equations in three-dimensional heterogeneous media. New hierarchical orthonormal basis functions on unstructured tetrahedral meshes are used for spatial discretization while Runge-Kutta methods for time discretization. A uniaxial perfectly matched layer (UPML) is employed to terminate the computational domain. Exponential convergence with respect to the order of the basis functions is observed and large parallel speedup is obtained for a plane-wave scattering model. The rapid decay of the out-going wave in the UPML is shown in a dipole radiation simulation. Moreover, the low frequency electromagnetic fields excited by a horizontal electric dipole (HED) and a vertical magnetic dipole (VMD) over a layered conductive half-space and a high frequency ground penetrating radar (GPR) detection for an underground structure are investigated, showing the high accuracy and broadband simulation capability of the proposed method.

AMS subject classifications: 65N30, 65F35, 65F15

Key words: Maxwell's equations, time domain, discontinuous Galerkin method, orthonormal polynomial basis functions, tetrahedral elements, UPML, parallelization.

1 Introduction

Numerical simulations of electromagnetic (EM) field have been widely used for understanding various EM phenomena and assisting the development of new technologies in

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areas such as electronic devices and advanced material design, optics, telecommunication, radar, geosciences, among many others, ranging from nano-structure to planet scale (e.g. [10, 14, 16, 23, 25, 35]). Finite element (FE) and finite difference (FD) methods have been popular numerical techniques in time domain EM field simulations (e.g. [11,27]) for decades. Both structured and unstructured meshes can be used for FE methods, allowing simulations in highly heterogeneous media and extremely complicated models. However, conventional nodal FE methods have difficulty handling the discontinuity of the EM field across the material interface, thereby unable to solve the first-order Maxwell's equations in heterogeneous media directly [11]. Vector FE methods are designed to cope with the discontinuous condition of the field, but traditional high order Nédélec vector basis functions [21] are not hierarchical and their implementation is more complicated. Though recent hierarchical version of Nédélec elements have been developed [33]. As to FD methods, such as Yee scheme [34], one of the major advantages is their straightforward implementation, but the use of cartesian grids greatly restricts their geometrical adaptability. Recently, discontinuous Galerkin methods (DGMs) are gaining popularity as an attractive numerical simulation tool, which not only possesses the flexibility to various problems and higher numerical accuracy and stability, but also has the capability to speedup the simulation with its intrinsic parallelism.

Reed and Hill [22] first introduced the DGM for solving linear neutron transport equations in 1973. Cockburn et al. [3–6] established the framework for DGMs to solve nonlinear time domain problems and DGMs have gained its recognition in diverse research areas later on. As a class of FE methods, DGMs adopt finite-element type meshes for spatial discretization and inherit the high geometrical adaptability of FE. Moreover, they allow the solutions to be discontinuous across the element interface by using discontinuous basis functions over the elements and defining numerical fluxes in the element interfaces, which differs from traditional continuous nodal FE methods. The local basis functions over one element are completely independent of those in neighboring elements, which offers inherent parallelism and allows the use of non-conforming meshes. There has been increasing researches on EM simulations using DGMs. Warburton [28] reported the use of DGM with polymorphic hp-finite elements for the EM scattering problem. Kopriva et al. [15] implemented a collocation form DG algorithm with non-conforming grids for scattering problems. Lu et al. [20] developed a DG scheme to solve the Maxwell's equations in 2-D dispersive and lossy media. Kabakian et al. [12] employed the DG method for broadband EM scattering simulation. Dolean et al. [7] adopted an implicit time integration scheme in DGM for EM simulation. Li et al. solved the Maxwell's equations using an implicit leap-frog DG method [17] and they further extended the leap-frog scheme to a nodal DGM [18]. Xie et al. proposed a space-time DGM for solving Maxwell's equations in free space [30] and applied the method to the simulation of meta-materials [31].

In this paper, we present a high-order DGM, with a recently developed orthogonal polynomial basis [32] over tetrahedral elements, for solving the time domain Maxwell's equations in three-dimensional heterogeneous media. The hierarchical orthonormal basis functions are chosen for spatial discretization not only for their high-order accuracy but