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Efficient Computation of Nonlinear Crest Distributions for Irregular Stokes Waves

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Abstract. This paper concerns the computation of nonlinear crest distributions for irregular Stokes waves, and a numerical algorithm based on the Fast Fourier Transform (FFT) technique has been developed for carrying out the nonlinear computations. In order to further improve the computational efficiency, a new Transformed Rayleigh procedure is first proposed as another alternative for computing the nonlinear wave crest height distributions, and the corresponding computer code has also been developed. In the proposed Transformed Rayleigh procedure, the transformation model is chosen to be a monotonic exponential function, calibrated such that the first three moments of the transformed model match the moments of the true process. The numerical algorithm based on the FFT technique and the proposed Transformed Rayleigh procedure have been applied for calculating the wave crest distributions of a sea state with a Bretschneider spectrum and a sea state with the surface elevation data measured at the Poseidon platform. It is demonstrated in these two cases that the numerical algorithm based on the FFT technique and the proposed Transformed Rayleigh procedure can offer better predictions than those from using the empirical wave crest distribution models. Meanwhile, it is found that our proposed Transformed Rayleigh procedure can compute nonlinear crest distributions more than 25 times faster than the numerical algorithm based on the FFT technique.

AMS subject classifications: 76B15, 65C50, 65C60, 65C05

Key words: Irregular Stokes waves, crest distribution, Transformed Rayleigh procedure, Fast Fourier Transform (FFT) technique.

1 Introduction

Knowing the wave crest probability distributions is of vital importance to the design and risk analysis of various kinds of ships and marine structures [1,2]. The probability

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distribution of the crest elevation (i.e., the highest elevation of an individual wave) must be established with care because it is used for the calculation of wave loads on a ship or an offshore platform. The extreme wave crest height is a key input parameter for deciding the bow height of a ship in order to avoid green water inundation. Furthermore, a proper statistical knowledge of the extreme wave crest height is essential to define a sufficient air gap under the deck of an offshore platform, and hence to ensure that a wave crest does not endanger the platform's integrity.

It is generally regarded that the wave crest height distributions obey the Rayleigh probability law in an ideal Gaussian random sea [3–7]. In the ideal Gaussian sea model the individual cosine wave trains superimpose linearly (add) without interaction, and therefore, the model is also called the linear sea model. However, waves in the real world are nonlinear [8–10]. Real waves show a small but easily noticed departure from a Gaussian surface. The crests are higher and steeper than expected from a summation of cosine waves with random phase, and the troughs are shallower and smaller [11, 12]. Meanwhile, it should also be pointed out that in the existing literature Lukomsky et al. [13, 14] and Lukomsky and Gandzha [15] have also developed some strongly nonlinear models applicable to extreme wave heights. Consequently, the application of the Rayleigh distribution to the wave crest heights will become invalid, and other more suitable methods should be applied to predict the distribution of wave crests for the nonlinear random model of the sea elevation.

Chakrabarti [1] mentions a model of wave crest height distribution of nonlinear ocean waves. This crest height distribution model was suggested four decades ago by Jahns and Wheeler by performing an empirical correction to the Rayleigh model of the crest height distribution. However, the previous research work of this author's research group has shown that such kind of empirical formulas will sometimes predict wave characteristic distributions that differ considerably from the true ones [16]. In order to overcome the weakness of the empirical approach for predicting the wave crest height distributions, numerical simulations can be performed either through solving the free-surface Euler equations [17, 18] or through Fast Fourier Transforming a wave spectrum. In this article, the author has developed a numerical algorithm based on the Fast Fourier Transform (FFT) technique [19–21] in order to compute the nonlinear crest distributions for irregular Stokes waves. For the numerical implementation of the nonlinear FFT simulation, the problem is divided into two processes, i.e. first generating the first order, linear (Gaussian) time histories with a specific wave spectrum at a specific location, for each of these the numerical algorithm then evaluates the full set of second order corrections. Therefore, the first order wave process, with N components at frequencies ω_n, gives rise to a total of N^2 corrections, spreading over all sum frequencies at frequencies $\omega_n+\omega_m$, and to another N^2 corrections over all difference frequencies $\omega_n-\omega_m$ [22, 23]. However, the nonlinear FFT simulation method as explained above will become very time consuming when N becomes large. Unfortunately, in a typical ocean engineering simulation project, the number N is usually of the order of 1000000 to 40000000. Therefore, there is a great need for a faster approach for computations of crest height distributions.