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Synchronization of Weighted Essentially Non-Oscillatory Methods

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Abstract. Weighted essentially non-oscillatory (WENO) methods have been developed to simultaneously provide robust shock-capturing in compressible fluid flow and avoid excessive damping of fine-scale flow features such as turbulence. Under certain conditions in compressible turbulence, however, numerical dissipation remains unacceptably high even after optimization of the linear component that dominates in smooth regions. Of the *non*linear error that remains, we demonstrate that a large fraction is generated by a "synchronization deficiency" that interferes with the expression of theoretically predicted numerical performance characteristics when the WENO adaptation mechanism is engaged. This deficiency is illustrated numerically in simulations of a linearly advected sinusoidal wave and the Shu-Osher problem [J. Comput. Phys., 83 (1989), pp. 32-78]. It is shown that attempting to correct this deficiency through forcible synchronization results in violation of conservation. We conclude that, for the given choice of candidate stencils, the synchronization deficiency cannot be adequately resolved under the current WENO smoothness measurement technique.

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1 Introduction

The detailed simulation of compressible turbulence requires numerical methods that simultaneously avoid excessive damping of spatial features over a large range of length scales and prevent spurious oscillations near shocks and shocklets (small transient shocks) through robust shock-capturing. Numerical schemes that were developed to satisfy these

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constraints include, among others, weighted essentially non-oscillatory (WENO) methods [2]. WENO schemes compute numerical fluxes using several different candidate stencils and form a final flux approximation by summing weighted contributions from each stencil. Thus they are nonlinear. Smoothness measurements cause stencils that span large flow field gradients to be assigned small relative weights so that a nearly discontinuous shock would provide a weight of almost zero to any stencil containing it. In smooth regions, the relative values of the weights are designed to be optimal by some gauge such as maximum order of accuracy or maximum bandwidth-resolving efficiency.

Jiang and Shu [3] cast the WENO methodology into finite-difference form and provide an efficient implementation of robust and high-order-accurate WENO schemes. Unfortunately, these schemes often generate excessive numerical dissipation for detailed simulations of turbulence, especially for large-eddy simulations (LES) [4]. WENO dissipation arises from two distinct sources: (i) the optimal stencil, which by itself describes a linear scheme, and (ii) the adaptation mechanism, which drives the final numerical stencil away from the optimal one. Bandwidth optimization can reduce the dissipation of the optimal stencil [5–7]; and Martín et al. [7] demonstrate that such a bandwidth-optimized symmetric WENO method indeed reduces numerical dissipation and provides accurate results for direct numerical simulations (DNS) of isotropic turbulence and turbulent boundary layers.

Nonetheless, engaging the nonlinear WENO adaptation mechanism still causes significant local dissipation that can negatively affect global flow properties. Though higher resolution compensates for this, in some cases adequately increasing the number of grid points is not feasible. There are two primary sources of nonlinear error: (i) the smoothness measurement that governs the application of WENO stencil adaptation and (ii) the numerical properties of individual candidate stencils that govern numerical accuracy when adaptation engages. Wang and Chen [8] have examined both sources for upwind-biased WENO methods in linearized problems; Ponziani et al. [9] have examined the second source for symmetric WENO methods in linear and nonlinear problems, including isotropic turbulence; and Henrick et al. [10] have examined the first source for upwind-biased WENO methods in linear and nonlinear problems. Additionally, Taylor et al. [11] have examined the first source for symmetric WENO methods in linear and nonlinear problems, including isotropic turbulence, and have introduced a linearly and nonlinearly optimized WENO method that allows accurate DNS of compressible turbulence with significantly reduced grid sizes [11, 12].

The purpose of this paper is to demonstrate that there exists a WENO "synchronization deficiency" that interferes with the expression of theoretically predicted candidate stencil properties and as a result generates excessive numerical dissipation through the second nonlinear error pathway described above. We furthermore attempt to correct this deficiency by exploring the possibility of forcible synchronization, and in the process we enumerate the several serious theoretical and practical obstacles that currently prevent an implementation of this approach that is both robust and broadly applicable. Section 2 briefly describes the WENO methodology. In Section 3, we introduce the mathematical