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On the Magneto-Heat Coupling Model for Large Power Transformers

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Abstract. This paper studies the magneto-heat coupling model which describes iron loss of conductors and energy exchange between magnetic field and Ohmic heat. The temperature influences Maxwell's equations through the variation of electric conductivity, while electric eddy current density provides the heat equation with Ohmic heat source. It is in this way that Maxwell's equations and the heat equation are coupled together. The system also incorporates the heat exchange between conductors and cooling oil which is poured into and out of the transformer. We propose a weak formulation for the coupling model and establish the well-posedness of the problem. The model is more realistic than the traditional eddy current model in numerical simulations for large power transformers. The theoretical analysis of this paper paves a way for us to design efficient numerical computation of the transformer in the future.

AMS subject classifications: 35K55, 35Q60, 78A25

Key words: Magneto-heat coupling model, eddy current problem, Maxwell's equations, radiative boundary condition.

1 Introduction

Large power transformers are broadly and increasingly used in countries undergoing rapid industrialization. Numerical simulation plays an important role in both computing iron loss and optimal design of large power transformers. It attracts great attention in both societies of computational electromagnetism and computational mathematics. The International Compumag Society has announced 34 benchmark problems or families of benchmark problems, called TEAM Workshop problems, to test electromagnetic analysis methods. Among them, the 21st family, Problem 21, was proposed to test numerical

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*Corresponding author. *Email addresses:* `lixujing@lsec.cc.ac.cn` (X. Li), `maosp@lsec.cc.ac.cn` (S. Mao), `yangkk@lsec.cc.ac.cn` (K. Yang), `zwy@lsec.cc.ac.cn` (W. Zheng) methods for computing iron loss in large power transformers [6]. It consists of 13 different models which represent various component accessories of a power transformer. Problem 21 is one of the most active TEAM Workshop problems and has promoted the development of numerical simulations of power transformer.

Power transformer consists of magnetic and nonmagnetic conductors, exciting coils, and cooling oils etc. The Ohmic heat and magnetic hysteresis lead to energy loss and can damage the devices of a power transformer. The oil is poured into and out of the power transformer to cool down the devices. Therefore, the system turns out to be the coupling between Maxwell's equations and the heat equation, or even more accurately, with fluid dynamics of the oil [17]. Traditional studies only consider quasi-static Maxwell's equations for TEAM Workshop Problem 21 and neglect the variation of electric conductivity caused by the temperature (see e.g. [7, 8, 12, 13, 16] and the references therein). In numerical simulation of power transforms, eddy current loss accounts for the major part of the total energy loss. The nonlinear dependency of the conductivity on the temperature is no longer negligible. The magneto-heat coupling model is more realistic and can yield better results (see [7, Chapter 3] and [14]). To the authors' best knowledge, there still lacks rigorous analysis in the literature on the well-posedness of the magneto-heat coupling model for power transformers. This paper is to fulfill the task.

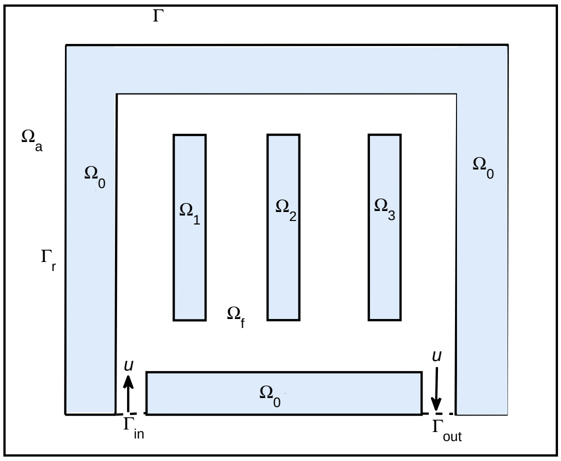

Figure 1: A 2D illustration of the problem geometry, $\Omega = \Omega_a \cup \Omega_f \cup \Omega_c$, $\Omega_c = \cup_{i=0}^{M} \Omega_i$.

The transformer system consists of the oil tank Ω_0, the cooling oil Ω_f contained by the tank, the conductors $\Omega_1, \cdots, \Omega_M$ which are immersed in the oil, and the air region Ω_a outside of the tank. The problem geometry is illustrated in Fig. 1. We assume that the oil tank is electrically conducting and both the air and the cooling oil are insulating. Then the conducting region and the non-conducting region are given respectively by

$$\Omega_c = \Omega_0 \cup \cdots \cup \Omega_M, \qquad \Omega_{nc} = \Omega_f \cup \Omega_a.$$