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Lattice Boltzmann Simulation of Steady Flow in a Semi-Elliptical Cavity

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Abstract. The lattice Boltzmann method is employed to simulate the steady flow in a two-dimensional lid-driven semi-elliptical cavity. Reynolds number (*Re*) and vertical-to-horizontal semi-axis ratio (*D*) are in the range of 500-5000 and 0.1-4, respectively. The effects of *Re* and *D* on the vortex structure and pressure field are investigated, and the evolutionary features of the vortex structure with *Re* and *D* are analyzed in detail. Simulation results show that the vortex structure and its evolutionary features significantly depend on *Re* and *D*. The steady flow is characterized by one vortex in the semi-elliptical cavity when both *Re* and *D* are small. As *Re* increases, the appearance of the vortex structure becomes more complex. When *D* is less than 1, increasing *D* makes the large vortexes more round, and the evolution of the vortexes with *D* becomes more complex with increasing *Re*. When *D* is greater than 1, the steady flow consists of a series of large vortexes which superimpose on each other. As *Re* and *D* increase, the number of the large vortexes increases. Additionally, a small vortex in the upper-left corner of the semi-elliptical cavity appears at a large *Re* and its size increases slowly as *Re* increases. The highest pressures appear in the upper-right corner and the pressure changes drastically in the upper-right region of the cavity. The total pressure differences in the semi-elliptical cavity with a fixed *D* decrease with increasing *Re*. In the region of the main vortex, the pressure contours nearly coincide with the streamlines, especially for the cavity flow with a large *Re*.

AMS subject classifications: 76M28, 76D05, 81T80

Key words: Semi-elliptical cavity, lid-driven flow, lattice Boltzmann method, boundary condition.

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1 Introduction

Recently, lid-driven cavity flows with different geometries have attracted much attention because they are related to a great deal of industrial applications [1]. Initially, for the sake of simplicity, many researchers only focused on the square cavity flow. In the past few decades, the square cavity flow, considered as a benchmark problem, has been subjected to intensive studies by different methods, such as finite difference method [2], finite element method [3], finite volume method [4], multigrid method [5], and lattice Boltzmann method (LBM) [6]. Some standardized conclusions about the square cavity flow have been obtained, which can be used to validate new numerical methods [7]. Later, it was found that compared with the square cavity flow, the rectangular cavity flow has a wider field of applications. Therefore, from then on many scholars began to study the rectangular cavity flow. The detailed analysis of the vortex structure and its evolutionary features for the rectangular cavity flow is presented in Refs. [8, 9], which show that there are significant differences for the rectangular cavity flow with various aspect ratios and Reynolds number (*Re*).

However, the realistic cavity flows are not limited in the rectangular cavity flow, and usually have more complex geometries. Therefore, interest in studying the lid-driven flows with complex geometries has grown significantly. Erturk and Dursun [10] studied the skew cavity flow with the skew angles varying from 15° to 165°. Some researchers investigated the lid-driven triangular cavity flow using finite different method [11, 12], finite element method [13], multigrid method [14], and LBM [15], respectively. McQuain et al. [16] investigated the trapezoidal cavity flow at *Re* of 1-500, later the trapezoidal cavity flow at *Re* of 100-15000 was studied in Ref. [17], in which a detailed analysis on the trapezoidal cavity flow for different *Re* and top angles was presented. In addition, Glowinski et al. [18] and Ding et al. [19] studied the characteristics of the vortex structure for the semi-circular cavity flow under different *Re*. Yang et al. [20] studied the semi-circular cavity flow with *Re* in the range of 5000-50000 and explained the transitional process of flow states. Mercan and Atalık [21] investigated the vortex structure of the arc cavity flow with various cross sections and *Re*. However, there are few attempts to study the semi-elliptical cavity flow. To our knowledge, there is only one publication on the semi-elliptical cavity flow [22]. Idris et al. [22] employed the stream-function-vorticity method to study the semi-elliptical cavity flow with the aspect ratios of 1/4, 1/3 and 3/8 and *Re* of 100-2000. By carefully investigating the literature presented by Idris et al. [22], we find that there are few numerical studies on the vortex structure in a semi-elliptical cavity with *Re* of greater than 2000 and the vertical-to-horizontal semi-axis ratio (D) of greater than 1. Therefore, the effects of *Re* and D on the vortex structure in a semi-elliptical cavity are as yet not completely understood.

The LBM is considered as a promising method to simulate fluid flows due to its many advantages, such as the simplicity of program, locality of computation, natural parallelism, better numerical stability and easiness in dealing with complex boundary [6, 23, 24]. Recently, the LBM has been used to investigate the flows in cavities with dif-