Commun. Comput. Phys.
doi: 10.4208/cicp.010415.110915a

Vol. **19**, No. 3, pp. 770-800
March 2016

Efficient Implementation of Smoothed Particle Hydrodynamics (SPH) with Plane Sweep Algorithm

Dong Wang[1], Yisong Zhou[2] and Sihong Shao[2,*]

[1] *State Key Laboratory of ASIC and System, School of Microelectronics, Fudan University, Shanghai 201203, China.*
[2] *LMAM and School of Mathematical Sciences, Peking University, Beijing 100871, China.*

Received 1 April 2015; Accepted (in revised version) 11 September 2015

Abstract. Neighbour search (NS) is the core of any implementations of smoothed particle hydrodynamics (SPH). In this paper, we present an efficient $\mathcal{O}(N\log N)$ neighbour search method based on the plane sweep (PW) algorithm with N being the number of SPH particles. The resulting method, dubbed the PWNS method, is totally independent of grids (i.e., purely meshfree) and capable of treating variable smoothing length, arbitrary particle distribution and heterogenous kernels. Several state-of-the-art data structures and algorithms, e.g., the segment tree and the Morton code, are optimized and implemented. By simply allowing multiple lines to sweep the SPH particles simultaneously from different initial positions, a parallelization of the PWNS method with satisfactory speedup and load-balancing can be easily achieved. That is, the PWNS SPH solver has a great potential for large scale fluid dynamics simulations.

AMS subject classifications: 76M28, 74F10, 35Q30, 68W05, 65D18

Key words: Smoothed particle hydrodynamics, meshfree method, neighbour search, plane sweep algorithm, Morton code, segment tree, quadtree, parallelization, dam break.

1 Introduction

Smoothed particle hydrodynamics (SPH) is a fully meshfree Lagrangian computational fluid dynamics (CFD) method developed independently by Lucy [1], Gingold and Monaghan [2] in 1977 for astrophysical studies. Due to its robustness in dealing with complex physical problems [3–9], SPH has since been successfully utilized to a large range of fields, such as ocean engineering [10], casting processes [11], semiconductor manufacturing [12] and so on. In SPH, the system is represented by discrete particles carrying their

*Corresponding author. *Email addresses:* `wangdong11@fudan.edu.cn` (D. Wang), `failed.zys@gmail.com` (Y. Zhou), `sihong@math.pku.edu.cn` (S. Shao)

own physical quantities. The quantity of each particle is interpolated from its neighbouring particles through a kernel function with compact support. Thus, efficient searching of particle neighbours is crucial for SPH performance. On account of the compact support property of the kernel function, only neighbour particles within its support domain are involved in calculating the force acting on a given particle. This significantly reduces the computing cost and serves as the start point of dozens of efficient neighbour search algorithms adopted in any SPH implementations over the $\mathcal{O}(N^2)$ direct traversing method (N is the number of SPH particles). An early survey of neighbour search algorithms was given in [13] and a collection of improved ones have been later proposed by the SPH community, including the grid-link-list method [14, 15], the hierarchical tree structured methods [16,17], the Hilbert-curve decomposition method [18,19], the Z-order curve indexing method [20], the spatial hashing method [21,22] and the adaptive-resolution cell lists method [23]. Among them, both the grid-link-list method and the hierarchical tree structured methods are much more popular.

In the grid-link-list method, the complexity of which is $\mathcal{O}(N\log N)$ [14,15], the entire domain is first divided into uniform grids sized in the radius of the support domain and then the neighbours of a given particle are searched only within its home and adjacent grids. This method is preferred in incompressible or weakly-compressible fluid problems for the smoothing length is often constant or varies in a small range. However, the grid-link-list method is not purely "mesh-free" for the search procedure highly depends on the grids. When the system consists of sparsely distributed particles (e.g., often encountered in adaptive particle refinement/derefinement [24,25]) or involves a large variation of the support domain, a great many grids may contain no particles, leading to an inefficient use of memory, and then the grid-link-list method is no longer suitable.

Another widely accepted idea for neighbour search comes from hierarchical tree structures and has been applied to SPH by many researchers using different algorithms. Hernquist et al. [16] incorporated the Barnes-Hut algorithm [26] into SPH calculation, where an octree is built hierarchically by dividing the entire domain into small cells and each particle is represented by a leaf node, and then the neighbour search is performed by descending from the top of the tree with a search cube checking whether a node is within the neighbouring area. Benz et al. [17] used a bottom-up tree structure, where the mutual neighbouring particles and tree nodes are organized in higher-level aggregate nodes recursively in a bottom-up hierarchy, and then a binary tree with neighbouring particles grouped together is built. These two tree methods are favoured in the astrophysics community for the ease of computing long-range gravitational forces, the mesh-less property and the flexibility for variational smoothing length. Although the complexity of these tree-based methods is reported to be $\mathcal{O}(N\log N)$ [16], it is shown that the logarithmic term can deteriorate into $\mathcal{O}(dN^{1-1/d})$ with d being the spatial dimension [27]. Moreover, parallelizing these tree-based methods is not an easy task and sometimes requires a redesign of the original algorithm [28].

The neighbour search in SPH is closely connected with the properties of smoothing kernels. Under most circumstances, the first choice of SPH community is the isotropic