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A High-Order Central ENO Finite-Volume Scheme for Three-Dimensional Low-Speed Viscous Flows on Unstructured Mesh

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Abstract. High-order discretization techniques offer the potential to significantly reduce the computational costs necessary to obtain accurate predictions when compared to lower-order methods. However, efficient and universally-applicable high-order discretizations remain somewhat illusive, especially for more arbitrary unstructured meshes and for incompressible/low-speed flows. A novel, high-order, central essentially non-oscillatory (CENO), cell-centered, finite-volume scheme is proposed for the solution of the conservation equations of viscous, incompressible flows on three-dimensional unstructured meshes. Similar to finite element methods, coordinate transformations are used to maintain the scheme's order of accuracy even when dealing with arbitrarily-shaped cells having non-planar faces. The proposed scheme is applied to the pseudo-compressibility formulation of the steady and unsteady Navier-Stokes equations and the resulting discretized equations are solved with a parallel implicit Newton-Krylov algorithm. For unsteady flows, a dual-time stepping approach is adopted and the resulting temporal derivatives are discretized using the family of high-order backward difference formulas (BDF). The proposed finite-volume scheme for fully unstructured mesh is demonstrated to provide both fast and accurate solutions for steady and unsteady viscous flows.

AMS subject classifications: 35Q30, 65Z05, 65M08, 65N08, 76M12, 76D05, 76G25

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1 Introduction

Computational fluid dynamics (CFD) has proven to be an important enabling technology in many areas of science and engineering. In spite of the relative maturity and widespread success of CFD in these areas, there is a variety of physically-complex flows which are still not well understood and are very challenging to predict with numerical methods. Such flows include, but are certainly not limited to, multiphase, turbulent, and combusting flows encountered in aerospace propulsion systems (e.g., gas turbine engines and solid propellant rocket motors). These flows present numerical challenges as they generally involve a wide range of complicated physical/chemical phenomena and scales.

Many flows of engineering interest are incompressible or can be approximated as incompressible to a high degree of accuracy, i.e. low-speed flows. Incompressible flows are challenging to solve numerically because the partial derivative of density with respect to time vanishes. As a result, the governing equations themselves are ill-conditioned. Various methods for solving the incompressible Navier-Stokes equations have been successfully developed to overcome this ill-conditioning [1, 2]. These include but are not limited to the pressure-Poisson [3, 4], fractional-step [5, 6], vorticity-based [7, 8], pseudo-compressibility [9], and characteristic-based methods [10, 11]. The equations governing fully-compressible flows have also been successfully applied to incompressible and low-speed flows by using preconditioning techniques [12–17]. The pseudo-compressible formulation [9, 18–25] is attractive because it is easily extended to three dimensions and applied in conjunction with high-order schemes. This method was originally referred to as the artificial compressibility method by Chorin [9], but Chang and Kwak [26] introduced the more accurate name "pseudo-compressibility method".

High-order methods have the potential to significantly reduce the cost of modelling physically-complex flows, but this potential is challenging to fully realize. As such, the development of robust and accurate high-order methods remains an active area of research. Standard lower-order methods (i.e, methods up to second order) can exhibit excessive numerical dissipation for multi-dimensional problems and are often not practical for physically-complex flows. High-order methods offer improved numerical efficiency for accurate solution representations since fewer computational cells are required to achieve a desired level of accuracy [27]. For hyperbolic conservation laws and/or compressible flow simulations, the main challenge involves obtaining accurate discretizations while ensuring that discontinuities and shocks are handled reliably and robustly [28]. High-order schemes for elliptic partial differential equations (PDEs) that govern diffusion processes should satisfy a maximum principle, even on stretched/distorted meshes, while also remaining accurate [29]. There are many studies of high-order schemes developed for finite-volume [28, 30–39], discontinuous Galerkin [40–49], spectral-difference/spectral-volume [50–54], flux reconstruction [55], and lifting collocation penalty methods [56, 57] on both structured and unstructured mesh. In spite of these many advances, there is still no consensus for a robust, efficient, and accurate scheme that fully deals with