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Variations on Hermite Methods for Wave Propagation

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Abstract. Hermite methods, as introduced by Goodrich et al. in [15], combine Hermite interpolation and staggered (dual) grids to produce stable high order accurate schemes for the solution of hyperbolic PDEs. We introduce three variations of this Hermite method which do not involve time evolution on dual grids. Computational evidence is presented regarding stability, high order convergence, and dispersion/dissipation properties for each new method. Hermite methods may also be coupled to discontinuous Galerkin (DG) methods for additional geometric flexibility [4]. An example illustrates the simplification of this coupling for Hermite methods.

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1 Introduction

The computational simulation of wave propagation is central to geophysical applications, such as seismic imaging and exploration, the modeling of seismic waves induced by earthquakes, and problems in structural acoustics. However, the numerical modeling of intermediate frequency waves is known to be challenging for many standard low-order methods, requiring a large number of points per wavelength to adequately resolve oscillatory behavior. Additionally, the simulation of propagating waves using low order methods is typically subject to significant non-physical (numerical) dissipation and dispersion. High order methods have the advantage of both rapid convergence

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and decreased numerical dissipation [10, 19] compared to low order methods, especially for problems in intermediate frequency wave propagation [14, 25]. High order methods also tend to have a high number of operations per data access, yielding a computational structure well-suited to modern computing architectures [21, 23, 24].

Hermite methods, as introduced by Goodrich et al. in [15], are high order methods for wave propagation which represent the solution using a piecewise polynomial basis by collocating the solution and its derivatives on a structured grid. Solution and derivative information at grid nodes is then used to reconstruct and evolve the solution in time on a staggered (dual) grid. Hermite methods are provably stable and high order accurate for hyperbolic equations, including problems with varying coefficients.

Furthermore, though the reconstruction step requires the access of non-local data at neighbor nodes, the computation of derivatives then depends only locally on the reconstructed data at each node. This is advantageous for high order or multi-stage timestepping methods compared to finite difference methods, where neighboring data must be accessed each time derivatives are approximated. This structure has also been noted to be well-suited for parallel implementations on modern architectures [2, 11]. Hermite schemes, which were initially introduced for Cartesian domains, have also been coupled with discontinuous Galerkin (DG) schemes for numerical simulations on complex geometries [4]. They have also been applied to problems in aeroacoustics [2], electromagnetics [4], and fluid dynamics [18].

The Hermite schemes of Goodrich et al. [15] are one instance of a broader family of methods involving collocation of the solution and its derivatives. Other methods in this family include shape-preserving methods [13, 28, 36] and jet schemes [5, 26, 33], which use Hermite interpolation in conjunction with semi-Lagrangian techniques to solve advective problems. These differ from the Hermite schemes discussed here in terms of the characteristic time evolution procedure; however, the analysis and stability of both Hermite and jet schemes both rely primarily on properties of Hermite interpolation under high order Sobolev seminorms.

Sections 1.1 and 2 present a generalized view of Hermite methods, and motivate new one-step Hermite schemes based on variations in the reconstruction procedure. These procedures also aim to simplify the implementation and coupling of Hermite and DG schemes [4]. Section 3 presents numerical experiments which confirm the high order convergence and stability of each method for the advection equation in one dimension. Section 4 extends each method to two space dimensions and includes numerical results for the two-dimensional advection and acoustic wave equations.

1.1 Time evolution

In this section, we introduce one-dimensional Hermite schemes for the approximation of an evolving solution and its derivatives at a collection of points over an interval $[a,b)\subset\mathbb{R}$. Each Hermite scheme presented has a timestep restriction based only on the domain of dependence for hyperbolic partial differential equations. For simplicity of presentation,