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Hamiltonian Analysis and Dual Vector Spectral Elements for 2D Maxwell Eigenproblems

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Abstract. The 2D Maxwell eigenproblems are studied from a new point of view. An electromagnetic problem is cast from the Lagrangian system with single variable into the Hamiltonian system with dual variables. The electric and magnetic components transverse to the wave propagation direction are treated as dual variables to each other. Higher order curl-conforming and divergence-conforming vector basis functions are used to construct dual vector spectral elements. Numerical examples demonstrate some unique advantages of the proposed method.

AMS subject classifications: 35Q61, 65P10, 65N30, 65N25

Key words: Maxwell eigenproblem, Hamiltonian system, symplectic eigenvalue analysis, dual variables, spectral element method.

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1 Introduction

2D Maxwell eigenproblem such as the waveguide analysis plays an important role in designing optical and microwave devices including microstrips and optical fibers. Popular finite element methods (FEM) [1–4] for this study are usually based on the second order wave equations with one variable (electric field **E** or magnetic field **H**). Once the FEM process is done and the discretized variable is obtained, numerical differentiation will be needed to calculate the other variable based on a fixed FEM mesh and the Maxwell's equations. Obviously the accuracy of the second variable obtained from this

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post-processing will be one order lower than that of the first variable, and this precision mismatch is undesirable for some applications requiring the values of both electric and magnetic fields simultaneously, e.g., the waveport implementation in time domain finite difference or finite element method. On the other hand, several schemes have been proposed simultaneously using both **E** and **H** as variables [5–8].

In this study we propose a Hamiltonian analysis and a dual vector spectral element method (SEM) for the 2D Maxwell eigenproblems. An electromagnetic problem is cast from the Lagrangian system with single variable into the Hamiltonian system [9], which is based on the transverse electric field and transverse magnetic field as dual variables to each other. Higher order curl-conforming and divergence-conforming vector basis functions based on the Gauss-Lobatto-Legendre (GLL) polynomials [10, 11] are employed to construct the dual vector SEM. The dual vector SEM discretization of an electromagnetic eigenproblem can achieve spectral accuracy with the increase of interpolation degrees of basis functions, and they can directly give the numerical solutions for both electric field and magnetic field at the same level of accuracy.

2 Hamiltonian system and dual variable variational principle

Conventional FEM analysis is usually based on one variable. In other words, it is described in the Lagrangian system. To develop the dual vector SEM in the Hamiltonian system, we need to start with the first order Maxwell's equations

$$\begin{cases} \nabla\times\mathbf{E}=-j\omega\mu\mathbf{H}, \\ \nabla\times\mathbf{H}=j\omega\epsilon\mathbf{E}. \end{cases} \tag{2.1}$$

Here excitation and dissipation terms are omitted without losing the generality of analyzing a eigenproblem. **E** is electric field and **H** is magnetic field. ϵ and μ denote the permittivity and permeability, respectively. The variational form for the above equations is

$$\Pi(\mathbf{E},\mathbf{H})=\int_\Omega\left[\mathbf{H}^*\cdot\nabla\times\mathbf{E}+\frac{j\omega\mu}{2}\mathbf{H}^*\cdot\mathbf{H}+\frac{j\omega\epsilon}{2}\mathbf{E}^*\cdot\mathbf{E}\right]d\Omega,\quad \delta\Pi=0, \tag{2.2}$$

where Ω denotes the area of this eigenproblem, and * is the complex conjugate operator. Performing the variational w.r.t. **E** and **H** independently in (2.2) will lead to the original equations (2.1). The proof steps are straightforward and will not be elaborated here.

To cast (2.2) into the Hamiltonian system we define the transverse electric and magnetic fields as dual variables [9]

$$\mathbf{q}=\mathbf{E}_t,\quad \mathbf{p}=\mathbf{H}_t\times\mathbf{z}, \tag{2.3}$$

where **z** denotes the direction of wave propagation. We also decompose the Nabla operator into transverse and longitudinal components

$$\nabla=\nabla_t+(\dot{})\mathbf{z}, \tag{2.4}$$