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Direct Calculation of Permeability by High-Accurate Finite Difference and Numerical Integration Methods

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Abstract. Velocity of fluid flow in underground porous media is 6∼12 orders of magnitudes lower than that in pipelines. If numerical errors are not carefully controlled in this kind of simulations, high distortion of the final results may occur [1–4]. To fit the high accuracy demands of fluid flow simulations in porous media, traditional finite difference methods and numerical integration methods are discussed and corresponding high-accurate methods are developed. When applied to the direct calculation of full-tensor permeability for underground flow, the high-accurate finite difference method is confirmed to have numerical error as low as 10^{-5}% while the high-accurate numerical integration method has numerical error around 0%. Thus, the approach combining the high-accurate finite difference and numerical integration methods is a reliable way to efficiently determine the characteristics of general full-tensor permeability such as maximum and minimum permeability components, principal direction and anisotropic ratio.

AMS subject classifications: 76S05

Key words: Finite difference method, numerical integration method, high accuracy, full tensor permeability, stokes.

1 Introduction

Fluid flow in subsurface porous media is quite slow. The flow rate is usually 6∼12 orders of magnitudes lower than the flow in pipes or channels. Thus, this kind of flow

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can usually be considered as quasi-steady flows at the pore scale. At the field scale, Darcy's law [5] was widely used to describe it and corresponding numerical methods were developed [6–10]. However, modeling of the flow by Darcy's law requires the permeability to be pre-determined [11]. The permeability is often considered as a symmetric full tensor, which contains 3 independent components in two-dimensional (2D) cases and 6 independent components in three-dimensional (3D) cases. This is because numerous practical problems make permeability measurements to be very difficult so that the experiments [12–21] and related models [22–30] are usually limited to the simplified assumptions of isotropy, homogenization or symmetry. These assumptions are practical for simple reservoirs. However, they may bury some important characteristics of real porous media which have complex geometric structures and largely reduce the accuracy of reservoir simulations [25,31,32]. Therefore, cancellation of the simplified assumptions using full-tensor permeability is a better approach. This approach does not require any assumptions on permeability. In our previous study, this new approach was demonstrated by direct-downscaling from Darcy scale to the pore scale described by Navier-Stokes equation [33]. However, the computational speed was not acceptable so that some important factors on permeability such as solid position, porosity and side length ratio were not studied. Thus, we expect to develop a new method with high accuracy and speed.

Since flows in reservoirs are quasi-steady and very slow, the convection terms are so weak that they can be neglected so that this kind of flow can also be simplified to be a steady-state Stokes flow. Therefore, the steady-state Stokes equation is adopted in this study instead of Navier-Stokes equation. The flow driven by gravity with periodic boundary condition used in the previous study [33] is also used in this study. Benchmark solutions of the full-tensor permeability are also important for studying characteristics of complex reservoirs so that they are provided by using this new approach. The details of the new numerical methods with high accuracy are introduced in Section 2 firstly and numerical results are discussed in Section 3.

2 High accurate numerical methods

2.1 High accurate methods for flow simulation

For 2D steady-state Stokes flow with constant fluid properties, we have the governing equations as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)+\mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+\rho g_x=0, \tag{2.2}$$

$$\mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)+\mu\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right)-\frac{\partial p}{\partial y}+\rho g_y=0, \tag{2.3}$$