## On the Spurious Mode Generation Induced by Spectral-Like Optimized Interpolation Schemes Used in Computational Acoustics

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**Abstract.** The present work constitutes a fraction of a more extensive study that is devoted to numerical methods in acoustics. More precisely, we address here the interpolation process, which is more and more frequently used in Computational Acoustics – whether it is for enabling multi-stage hybrid calculations, or for easing the proper handling of complex configurations via advanced techniques such as Chimera grids or Immersed Boundary Conditions. In that regard, we focus on high-order interpolation schemes, so as to analyze their intrinsic features and to assess their effective accuracy. Taking advantage of specific insights that had been previously achieved by the present authors regarding standard high-order interpolation schemes (of centered nature), we here focus on their so-called spectral-like optimized counterparts (of both centered and noncentered nature). The latter spectral-like optimized schemes are analyzed thoroughly thanks to dedicated theoretical developments, which allow highlighting better what their strengths and weaknesses are. Among others, the various ways such interpolation schemes can degrade acoustic signals they are applied to are carefully investigated from a theoretical point-of-view. Besides that, specific criteria that could help in optimizing interpolation schemes better are provided, along with generic rules about how to minimize the signal degradation induced by existing interpolation schemes, in practice.

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**Key words**: Noncentered interpolation, centered interpolation, high-order, spectral-like optimization, spurious modes generation, computational acoustics.

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## 1 Introduction

Nowadays, many fields of research focus on the modeling of noise phenomena, which finds plenty of applications within numerous domains. Indeed, adequately modeling and predicting noise phenomena can help improving existing technologies, such as reducing the noise annoyances by industrial products (e.g. aircraft, trains or cars, wind turbines, computers). It can also help optimizing specific acoustic-based devices, such as those used for non destructive control, non intrusive medical imaging, seismic monitoring, and so on. Finally, it can facilitate the emergence of new technologies, e.g. alternative and innovative acoustic-based devices (e.g. sensorless tactile screens, ultra-sound medical curative techniques). On another hand, since acoustics is a complex discipline, researchers are now bounded to make an intensive use of numerical simulation, which constitutes a powerful means of investigations, when coming in complement to the experimentation.

This, however, requires the continuous development and improvement of modeling and solving techniques that can account for all mechanisms underlying the noise generation and/or propagation phenomena. Indeed, noise finds its origin in numerous source mechanisms such as structural vibrations, fluid motions, flow interactions with structures, gas combustion or explosions. Once they have been generated by these sources, acoustic waves propagate within the surrounding environment, which is generally constituted with one or several media of various complexity (e.g. comprising solid bodies and/or medium heterogeneity). During such propagation phase, acoustic waves may thus be submitted to numerous and important alterations in terms of amplitude, phase or frequency, which all result from mechanisms as diverse as reflection and diffraction effects by solid structures, convection by fluid motions, refraction and diffusion by the medium heterogeneity (e.g. turbulence, temperature gradients), absorption by the medium viscosity, etc.

Many of the acoustic generation processes and most of the acoustic propagation mechanisms are relevant to the physics of fluid dynamics, and can thus be simulated by numerically solving the Navier-Stokes equations. However, the specificity of acoustic phenomena implies that such solving is achieved following very precise strategies. In particular, this is what led to the emergence of Computational AeroAcoustics (CAA), a scientific discipline that rapidly progressed over the past twenty years. Among others, such discipline promoted the development of highly-accurate numerical schemes, whose minimal dissipation and dispersion properties are mandatory for acoustic waves are correctly simulated over long propagation distances and time durations [1]. As an illustration, given their higher accuracy and, thus, ability to simulate wave-like phenomena on coarser grids [2], high-order Finite-Difference (FD) methods have emerged as a valuable alternative to more traditional low-order methods that are usually employed in other disciplines (e.g. Computational Fluid Dynamics, CFD).

At the present time, however, and despite of the continuous development of computational tools and resources, it is still very challenging to simulate aero-acoustic problems