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An Adaptive Perfectly Matched Layer Method for Multiple Cavity Scattering Problems

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Abstract. A uniaxial perfectly matched layer (PML) method is proposed for solving the scattering problem with multiple cavities. By virtue of the integral representation of the scattering field, we decompose the problem into a system of single-cavity scattering problems which are coupled with Dirichlet-to-Neumann maps. A PML is introduced to truncate the exterior domain of each cavity such that the computational domain does not intersect those for other cavities. Based on the a posteriori error estimates, an adaptive finite element algorithm is proposed to solve the coupled system. The novelty of the proposed method is that its computational complexity is comparable to that for solving uncoupled single-cavity problems. Numerical experiments are presented to demonstrate the efficiency of the adaptive PML method.

AMS subject classifications: 65N30, 65N50

Key words: Uniaxial perfectly matched layer, multiple cavity scattering, adaptive finite element, a posteriori error estimate.

1 Introduction

We propose and study a uniaxial perfectly matched layer (PML) method for solving the multiple cavity scattering problems

$$\Delta u + k^2 u = 0 \qquad \text{in } D \cup \mathbb{R}^2_+, \tag{1.1a}$$

$$u = 0 \qquad \text{on } \Gamma^c \cup S, \tag{1.1b}$$

$$[u] = \left[\frac{\partial u}{\partial x_2}\right] = 0 \quad \text{on } \Gamma_D, \tag{1.1c}$$

$$\lim_{r=|x|\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial r} - \mathbf{i} k_0 u^s\right) = 0, \tag{1.1d}$$

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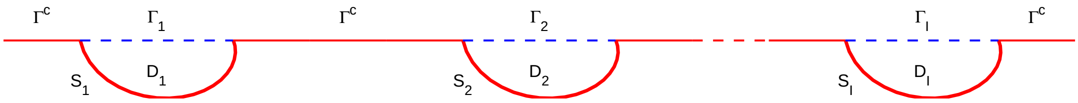

Figure 1: An illustration for the setting of multiple cavity scattering problem.

where u and u^s are the total field and the scattering field respectively and $\mathbf{i}$ is the imaginary unit. The radiation condition (1.1d) is imposed in the upper half space $\mathbb{R}^2_+$ where

$$\mathbb{R}^2_\pm = \{(x_1, \pm x_2)\,|\,x_1 \in \mathbb{R},\, x_2 > 0\}, \qquad \Gamma = \partial\mathbb{R}^2_+ = \{(x_1, 0)\,|\,x_1 \in \mathbb{R}\}.$$

The region $D \subset \mathbb{R}^2_-$ consists of well-separated cavities, $D = \cup_{i=1}^I D_i$, which are bounded and have Lipschitz boundaries. For each cavity, say D_i, S_i denotes the cavity wall and Γ_i denotes the cavity aperture (see Fig. 1 for a simple illustration), namely,

$$\partial D_i = \Gamma_i \cup S_i, \qquad 1 \leq i \leq I.$$

For convenience we let

$$S = \cup_{i=1}^I S_i, \qquad \Gamma_D = \cup_{i=1}^I \Gamma_i, \qquad \Gamma^c = \Gamma \backslash \Gamma_D.$$

The wavenumber $k(\boldsymbol{x})$ is assumed to be constant in the upper half plane

$$k = k_0 \qquad \text{in } \mathbb{R}^2_+, \tag{1.2}$$

but may be inhomogeneous inside cavities. Let $u_\pm$ denote the limits of u as the argument goes to Γ from above and below respectively. Then the jump of u across Γ is defined by

$$[u] = u_+ - u_- \quad \text{on } \Gamma.$$

The scattering of cavities in the infinite ground plane is of great importance for its industrial and military applications. There are plenty of papers that study the scattering problems by cavities both in the engineering community and the mathematical community. In [21, 25], Jin et al. studied high-order finite element approximations to the scattering problem by deep cavities. Based on Fourier's transform, Ammari et al. [1, 2], Bao and Sun [6], Van and Wood [29] studied nonlocal transparent boundary conditions on the open aperture of the cavity. A mode matching method was proposed by Bao et al. [5, 8] for solving electromagnetic scattering problem by large cavities. For scattering problems by overfilled cavities, one can not restrict the computational domain to cavities any more. Wood [31] and Li et al. [24] introduced an artificial boundary condition on a semicircle or hemisphere and developed numerical methods for dealing with the scattering by overfilled cavities. We also refer to [4, 16, 18, 19, 22, 30, 32–34] and the references therein for various numerical investigations into cavities scattering problems, such as finite difference method, finite element method, boundary element method and hybrid methods.