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An Immersed Interface Method for Axisymmetric Electrohydrodynamic Simulations in Stokes flow

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Abstract. A numerical scheme based on the immersed interface method (IIM) is developed to simulate the dynamics of an axisymmetric viscous drop under an electric field. In this work, the IIM is used to solve both the fluid velocity field and the electric potential field. Detailed numerical studies on the numerical scheme show a second-order convergence. Moreover, our numerical scheme is validated by the good agreement with previous analytical models [1,31,39], and numerical results from the boundary integral simulations [17]. Our method can be extended to Navier-Stokes fluid flow with nonlinear inertia effects.

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1 Introduction

Under a direct current (DC) electric field an electrohydrodynamic flow is induced both inside and around a viscous drop [28,37,39]. Depending on the electrical properties of the fluids the viscous drop can deform into a prolate (oblate) spheroid with its major axis parallel (perpendicular) to the electric field. Such electrohydrodynamics of a viscous drop has a wide range of applications in micro-fluidic systems [36,45], from drop manipulation by electro-wetting [6] to enhanced mixing inside and on a drop in a Stokes

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flow [2, 11, 32, 44]. Several theoretical models and numerical schemes have been developed to study the drop dynamics under a DC electric field. Theoretical studies are often restricted to nearly spherical [1, 39, 42] or spheroidal shapes [4, 31, 46]. Consequently these models can only capture the drop electrohydrodynamics up to a moderate electric field, and cannot predict the extreme drop elongation, lobed steady drop shape and several break-up modes under a strong DC field observed in experiments [13, 14] and direct numerical simulations. Various numerical schemes have been developed to simulate electrohydrodynamic (EHD) flow with certain limitations. For example, the boundary integral method [7, 17, 38] cannot be easily extended to different configurations such as solving the full Navier-Stokes equations, or accounting for charge effects in the bulk fluid. Sharp methods such as the ghost-fluid method [33, 35] are generally first-order accurate. Tomar *et al.* [41] used the volume of fluid (VOF) method, where jumps in electrical and fluid properties across the drop interface are smoothed out in a transition region around the moving interface. A direct consequence of the smoothing is the reduction in the order of accuracy. The finite element method [10] is too computationally expensive for deforming/moving interfaces.

To investigate the electrodeformation of a viscous drop with inertia effects (finite Reynolds number), recently Hu *et al.* [15] developed a hybrid numerical scheme, using the immersed boundary (IB) method for the incompressible Navier-Stokes flow (finite Reynolds number) and the augmented immersed interface method (AIIM) for the electric potential. The authors treated the electric Maxwell stress as an interfacial force to facilitate a unified immersed boundary framework, and illustrated first-order convergence for the electric force calculations. In addition this code can efficiently simulate large drop deformation under both an electric field and a shear flow.

Based on results in [15], we have developed an immersed interface method (IIM) for axisymmetric electrohydrodynamics of a viscous drop in the present study. Both the electric and the fluid equations are solved within the immersed interface framework with axisymmetry. The fluid solver is based on an axisymmetric immersed interface algorithm [24]. The electric solver is an AIIM modified from [15] to account for the axisymmetry of the drop interface.

The IIM was initially developed to solve elliptic equations with discontinuous coefficients and/or singular sources [20]. The basic idea behind the IIM is to modify the finite difference discretization of the discretized governing equations near irregular grid points by adding correction terms that depend on the boundary conditions at an interface. The IIM differs from the immersed boundary method (IB method) [34], another popular method for solving similar problems, in that the IIM does not regularize the singular forces at the interface. Instead, it captures the interfacial discontinuity in a sharp manner, and yields a second-order convergence rate. While second-order IB methods exist [5, 12, 19, 30], the second-order accuracy is reduced to first order for problems that involve an infinitely sharp interface [29], such as the drop interface that we consider here. Therefore in this work we use IIM with consistent order of accuracy (second-order) for both the electric field and the fluid flow. In addition, our IIM algorithm is developed for