Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2018-0068

Vol. **26**, No. 1, pp. 206-232
July 2019

Advection-Enhanced Gradient Vector Flow for Active-Contour Image Segmentation

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Received 17 March 2018; Accepted (in revised version) 23 August 2018

Abstract. In this paper, we propose a new gradient vector flow model with advection enhancement, called advection-enhanced gradient vector flow, for calculating the external force employed in the active-contour image segmentation. The proposed model is mainly inspired by the functional derivative of an adaptive total variation regularizer whose minimizer is expected to be able to effectively preserve the desired object boundary. More specifically, by incorporating an additional advection term into the usual gradient vector flow model, the resulting external force can much better help the active contour to recover missing edges, to converge to a narrow and deep concavity, and to preserve weak edges. Numerical experiments are performed to demonstrate the high performance of the newly proposed model.

AMS subject classifications: 68U10, 65K10

Key words: Image segmentation, active contour, gradient vector flow, external force.

1 Introduction

The technique of snake (also called active contour) was first introduced by Kass *et al.* [14] for image segmentation and it has been widely used in many applications of computer vision and image processing, such as edge detection, shape recognition, and object tracking etc., see e.g., [11,14,17,21,28]. Roughly speaking, a snake is represented by a contour that can deform to achieve boundary extraction of the desired object through an energy-minimization process. The contour-minimizing energy basically consists of two parts,

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in which the internal energy part, defined by the snake itself, is designed to control the smoothness and tightness of the snake, while the external energy part, represented by a certain image quantity, is used to drive the snake to approach the desired features in the image such as object edges. By using the calculus of variations, a minimizer of the energy functional satisfies a force balance equation which is also consisting of two parts, the internal force and the external force. When the internal and external forces achieve balance, the energy achieves a minimum value.

According to the ways of implementing the snake, there are basically two types of active contours. One is the parametric active-contour [9, 10, 14, 18, 25, 26], where contours are explicitly represented as parameterized curves, and the other is the geometric active-contour [4–6, 20], which is largely inspired by level set methods and represents curves implicitly. In general, the former has much lower computational complexity, so its implementation is efficient in convergence speed. Nevertheless, the latter is capable of handling changes in the topology of contours so that the snake can split or merge naturally during the evolution process for dealing with images with multiple objects.

In this paper, we will focus on the parametric active-contours for image segmentation. Although the parametric snake is able to perform flexible deformations, some major difficulties such as small capture range, deep and narrow concavity convergence and weak-edge leakage still remain and need to be addressed further [8,15,18,19,22,30,34]. To overcome these limitations, the external force field, which is usually written as the gradient of a potential function, plays an important role and it has been extensively improved over the years. In 1998, Xu and Prince [32] proposed the gradient vector flow (GVF), which diffuses the gradient vector field (i.e., external force field) to enlarge the capture range and improves the convergence for entering concavities. However, the snake using GVF external force is not able to go into long and thin indentations (LTIs for short) [23,33] and may cause leakage problems [32]. Due to this reason, Xu and Prince then developed a generalized version of the GVF snake, called the generalized GVF (GGVF) [33], by adding two spatially varying weighting parameters to improve the convergence into LTIs and protect weak edges. Afterwards, Ning *et al.* [23] introduced an improved external force field for snakes, called the GVF in normal direction (NGVF for short). By decomposing the Laplacian operator and adopting only the normal component, the NGVF snake can achieve a faster convergence speed towards concavities, but the convergence capability for LTIs has not been significantly improved. On the other hand, Wang *et al.* [29] studied another new external force field, called the normally biased GVF (NBGVF). It mainly adopts the tangential component of diffusion to preserve weak edges, and then adds the normal component as a biasing weight to maintain some desirable properties of GVF and NGVF. Recently, to take care of both weak-edge preserving and LTI convergence, Wu *et al.* [30] investigated an adaptive diffusion flow (ADF) field by adaptively adopting a harmonic hypersurface minimal functional and an infinity Laplacian functional. The ADF snake possesses several good properties including weak-edge preserving, concavity convergence, and noise robustness. In summary, all the above methods mainly employ diffusion to act on the gradient vector field, in the direction of tangential, normal, or both.