## **Computational Optimal Design of Random Rough Surfaces in Thin-Film Solar Cells**

Gang Bao<sup>1</sup>, Yanzhao Cao<sup>2</sup>, Junshan Lin<sup>2,\*</sup> and Hans Werner Van Wyk<sup>2</sup>

<sup>1</sup> Department of Mathematics, Zhejiang University, China.
<sup>2</sup> Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849, USA.

Received 16 January 2018; Accepted (in revised version) 19 June 2018

**Abstract.** Random rough textures can increase the absorbing efficiency of solar cells by trapping the optical light and increasing the optical path of photons. In this paper, we are concerned with optimal design of random rough surfaces in thin-film solar cells. We formulate the design problem as a random PDE constrained optimization problem and employ gradient-based methods for solving the problem numerically. To evaluate the gradient of the objective function, the Monte-Carlo method is used for sampling the probability space and the adjoint state method is employed to calculate the gradient at each sample. Numerical examples are shown to test the efficiency of the proposed algorithm. It is demonstrated that optimally obtained random textures yield an enormous absorption enhancement and a higher photon absorptance than that of existing random textures.

**AMS subject classifications**: 35J05, 35Q60, 49Q10, 65C05, 65C30 **Key words**: Optimal design, random rough surface, solar cell, Helmholtz equation.

## 1 Introduction

Photovoltaics (PV), which directly convert solar energy into electricity, offer a practical and sustainable solution to the challenge of meeting the increasing global energy demand. A typical photovoltaic system employs solar panels, each consisting a number of solar cells to generate electrical power from sun light that can be used to power equipment or to recharge a battery [21]. One representative cell configuration is the so-called *thin-film solar cell*, which is made of hydrogenated amorphous silicon (a-Si:H) and microcrystalline silicon ( $\mu$ c-Si:H). A typical a-Si:H cell structure is shown in Fig. 1, where the

http://www.global-sci.com/

<sup>\*</sup>Corresponding author. *Email addresses:* bao@math.zju.edu.cn (G. Bao), yzc0009@auburn.edu (Y. Cao), jz10097@auburn.edu (J. Lin), hzv0008@auburn.edu (H. W. Van Wyk)



Figure 1: A schematic plot of thin-film solar cells.

intrinsic a-Si:H is the absorbing layer that has a thickness of a few hundred nanometers. Compared to traditional crystalline silicon solar cells, thin-film cells offer several distinctive features such as much smaller thickness, low cost in production, and special optical properties of a-Si:H and  $\mu$ c-Si:H [26].

The mass production of stable a-Si:H cells requires that the maximum thickness of their light absorbing layer is often limited to about 300 nm [26]. Such a layer is sufficiently absorptive at smaller optical wavelengths and all the incoming light in that frequency band can be effectively absorbed. However, at larger wavelengths (typically > 600 nm), a-Si:H is poorly absorptive and most photon energy escapes. Consequently, the overall efficiencies of thin-film solar cells are low, and their optical structures have to be engineered in a way so as to increase the absorption and enhance their performance.

There exist various ways to increase the absorption efficiency of solar cells, for instance, antireflection coating, fluorescent dyes, dielectric gratings, photonic crystals, and plasmonic nanoparticles, etc [3, 4, 7, 13, 28]. However, it is still unclear whether such design will find their way into commercial photovoltaic devices because of their high costs. Another way to increase the efficiency of photon absorption is using randomly nanotextured interfaces to trap the optical light [1, 11, 12, 17, 27]. The randomly textured surfaces lower the reflection losses at the entrance facet and scatter the light, thereby increasing the optical path of each photon in the solar cell. In thin-film solar cells, this is usually achieved by texturing the surface of the transparent conductive oxide (TCO) layer as shown in Fig. 1 (see also [22]). Very importantly, in practice the control and fabrication of the TCO interface in a random manner can be achieved at very low cost by controlling the deposition parameter of TCO films sputtered on glass substrates [18, 23]. Such