

## Numerical Study of Turbidity Current over a Three-Dimensional Seafloor

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**Abstract.** Turbidity current over a Gaussian-bump is investigated numerically using the upwinding combined compact difference (UCCD) scheme and the immersed boundary (IB) method in Cartesian grids. In the prediction of lock-exchange gravity-driven flow motion, the initial discontinuous concentration field is smoothed to avoid numerical oscillation by solving the Hamilton-Jacobi equation. The UCCD spatial scheme with sixth-order accuracy which introduces less dispersion errors is then used to discretize advection and diffusion terms in the calculation of concentration transport equation. Direct forcing IB method is employed to treat solid object bumps in the fluid flow. The incompressible Navier-Stokes solutions are obtained through the projection method. Analysis of the smoothing procedure, grid sensitivity and the effect of the Schmidt numbers is performed for the turbidity current problem to validate the proposed numerical algorithm, which is shown to be capable of accurately demonstrating their results. Finally, several problems of turbidity current over a three-dimensional seafloor are investigated. The front locations of the currents interacting with the bump are predicted under different Reynolds numbers. Also, the current properties, namely the suspended particle mass, sedimentation rate and energy budget, are compared with the available numerical results.

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**Key words:** Turbidity current, upwinding combined compact difference scheme, immersed boundary method, concentration transport equation, sedimentation rates, energy budget.

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## 1 Introduction

The density difference between flow and its ambient environment results in the formation of gravity currents [1–4]. Among all types of gravity currents, turbidity current, has a tendency to be slowed down earlier due to settlement-induced energy loss of particles. Turbidity current is ubiquitous in various natural processes. For example, the turbidity currents flowing down continental shelves can suddenly release dense gases into the atmosphere. Besides, turbidity currents can interact with seafloors of various shapes, which may form complex topographical features including fans and lobes, gullies, levees and sediment waves [5–7]. Further investigation is needed to study the dynamical mechanism of turbidity currents.

Laboratory experiments have been conducted to investigate evolution of turbidity currents [8]. A high resolution scheme is then adopted to investigate gravity currents in a rectangular domain with a varying slope [9]. The lock-exchange gravity current propagating along a flat bottom is studied by a two-way coupled Euler-Lagrange model proposed by Chou et al. [10]. Their numerical simulations have revealed more detailed features of particle-laden flow concerning front location and deposited mass. For more information about the theoretical formulation and application of two-way coupled Euler-Lagrange models, one can refer to [11, 12]. Large Eddy Simulation (LES) is applied to study the evolution of gravity currents and its interaction with structures on complex 3-D seafloors under high Reynolds number conditions [13]. Direct Numerical Simulation (DNS) is also applied to investigate the interaction of turbidity currents with bumps of various shapes. Investigation shows that mixing of gravity currents with the ambient environment is not as intensive as that of turbidity currents because of the difference in particle settling velocity [5]. Refs. [14–18] discussed the features of turbidity current and its accompanying instability mechanism in detail using high resolution schemes.

High resolution schemes, i.e., ENO (Essentially non-oscillatory) and WENO (Weighted essentially non-oscillatory) [19, 20], usually adopt different nonlinear adaptive procedures to obtain locally smoothest stencils, to suppress numerical oscillations across physical discontinuities. It is well known that ENO and WENO schemes are over-dissipative in turbulence simulation because of the excessive dissipation introduced. Continuous efforts have been dedicated to WENO schemes to expand its application area. Latest development in WENO schemes can be found in [21].

Different from high resolution schemes, compact schemes receive in-depth investigation in DNS of Navier-Stokes simulation thanks to their advanced capability in providing high accuracy solutions [22]. Besides, compact schemes are less costly and more accurate than explicit finite difference schemes. Centered compact difference schemes were first proposed by Lele [23] in 1970s. However, due to its zero-dissipation property, centered compact difference schemes may induce high-frequency oscillations in smooth flow regions. In order to eliminate this high-frequency oscillation, artificial viscosity should be included. Upwind compact schemes contain inherent artificial viscosity, which makes it more stable than centered compact schemes [24–29]. It should be noted, however, that, it