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A Solution Structure-Based Adaptive Approximate (SSAA) Riemann Solver for the Elastic-Perfectly Plastic Solid

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Abstract. The exact Riemann solver for one-dimensional elastic-perfectly plastic solid has been presented in the previous work [S. Gao and T. G. Liu, Adv. Appl. Math. Mech., 9(3), 2017, 621-650], but its iterative process of finding nonlinear equation solution is time-consuming. In this paper, to enhance the computational efficiency of the exact Riemann solver and provide a more practical Riemann solver for actual implementation, we design a non-iterative solution structure-based adaptive approximate (SSAA) Riemann solver for one-dimensional elastic-perfectly plastic solid. Judging the solution structure adaptively and then solving the Riemann problem with corresponding solution structure non-iteratively can shorten the computing time and meanwhile guarantee the correctness of the final result. Numerical performance tests manifest that the exact Riemann solver is indeed time-consuming and the ordinary approximate Riemann solver with fixed three-wave solution structure is of great error, whereas the SSAA Riemann solver is of both efficiency and accuracy. Error estimation further indicates that the SSAA Riemann solver has at least second-order accuracy to approach the exact solution of the states in the star region.

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Key words: Riemann solver, elastic-plastic solid, ghost fluid method, nonlinear hyperbolic system.

1 Introduction

In the current computational fluid dynamic (CFD) field, various Riemann solvers have been widely adopted in the Godunov-type scheme [1–5] to predict numerical fluxes and

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in the ghost fluid-type method [6–12] to define ghost fluid states. Among them, the exact Riemann solver [13] is the most fundamental one, which judges the solution type firstly and then utilizes the bisection method or Newton-Raphson method to iterate the root of nonlinear equation regarding to the pressure in the star region. However, this iteration process is rather time-consuming especially when the exact Riemann solver is invoked frequently. For that reason, researchers prefer to employ approximate Riemann solvers rather than utilize the exact one in practical implementation. Hence, a number of approximate Riemann solvers have been designed in different ways to speed up the solving procedure as much as possible, under the premise that the numerical result is acceptable. But, there is an underlying designing principle of these approximate Riemann solvers, which is that their solution structures all largely depend on the exact solution structure of the specific Riemann problem.

For one-dimensional compressible fluid Riemann problem, it is acknowledged that its exact solution structure is typically three-wave. Thus a reasonable solution structure of the approximate Riemann solver should be three-wave as well. Following this thought, three-wave iterative approximate Riemann solvers like double-rarefaction Riemann solver, double-shock Riemann solver and their variants [14] can be easily derived from the exact Riemann solver. The superiority of these iterative three-wave approximate Riemann solvers is that they can skip the judgement of solution type, which is a necessary step in the exact Riemann solver. However, the deficiency is that they still need the iterative process to locate the root of nonlinear equation regrading to the pressure in the star region. To further develop a three-wave approximate Riemann solver without iteration, the primitive variable Riemann solver (PVRS) [15] was proposed. PVRS is constructed by the linearized characteristic equations in terms of the primite variable hyperbolic system. It owns outstanding performance on computation efficiency and is able to compute numerical results much more rapidly than the fore-mentioned iterative approximate Riemann solvers. Besides the PVRS, there is another frequently-used non-iterative approximate Riemann solver, the HLL (Harten-Lax-van Leer) Riemann solver [16]. It can predict the numerical flux directly without needing to calculate the states in the star region in advance. But it should be noted that the solution structure of HLL Riemann solver is only two-wave, where the linear degenerated contact discontinuity is omitted. Such a simplification on solution structure causes the HLL Riemann solver tending to introduce a relatively large error when calculates the states in the star region. And when the HLL Riemann solver is applied in the Godunov-type method, its simulation result inclines to give obvious numerical dissipations near the contact discontinuity in the density profile owing to its imperfect solution structure. To compensate this offset, the effect of contact discontinuity has to be taken into account and a three-wave non-iterative HLLC (C standing for the Contact discontinuity) approximate Riemann solver [17] was put forward. Hence, from the above construction manner of approximate Riemann solvers for the compressible fluid, it can be found that a high-accuracy approximate Riemann solver should ensure its solution structure is uniform with the exact solution structure and any simplification on the solution structure (for example, the two-wave simplification of the